Solution 1.

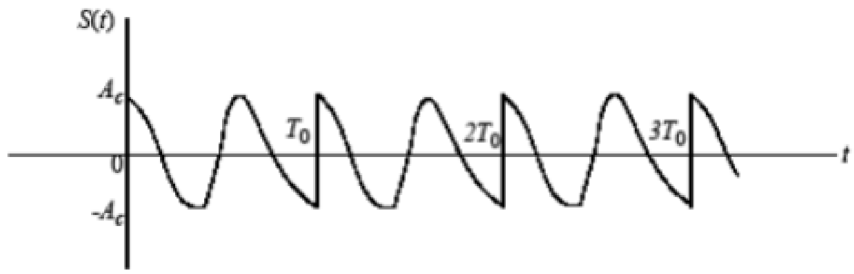
根據頻率來畫圖。瞬時頻率求法：$f_i(t)=\frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$

(1). $s_{PM}(t)=A_c\cos\left[2\pi f_c t+k_p m(t)\right]$

instantaneous angle $\theta_i(t)=2\pi f_c t+k_p m(t)$

instantaneous frequency $f_i(t)=f_c+\frac{k_p}{2\pi}\frac{dm(t)}{dt}$

由上式可以知道 PM signal 的頻率跟 $m(t)$ 的斜率呈現正比關係。(訊號斜率為 1，故頻率不變。)

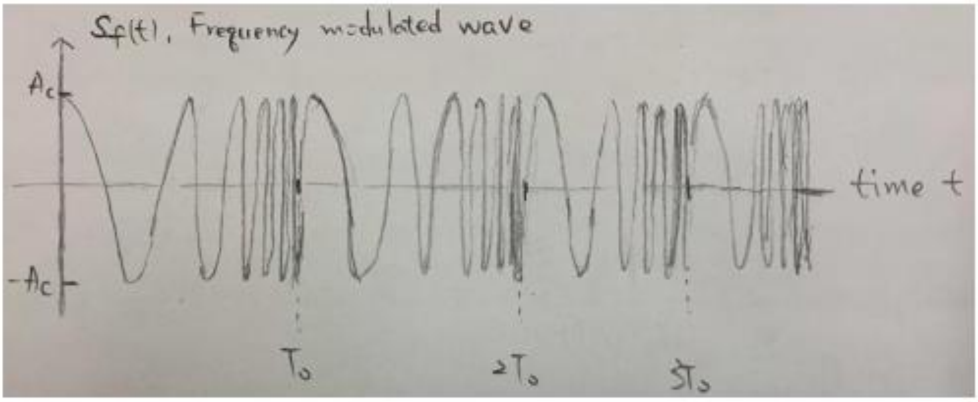


(2). $s_{FM}(t)=A_c\cos\left[2\pi f_c t+2\pi k_f\int_0^t m(\tau)d\tau\right]$

instantaneous angle $\theta_i(t)=2\pi f_c t+2\pi k_f\int_0^t m(\tau)d\tau$

instantaneous frequency $f_i(t)=f_c+k_f m(t)$

由上式可以知道 FM signal 的頻率跟 $m(t)$ 振幅呈現正比關係。(訊號愈來愈大，頻率也會愈來愈大)



Solution 2.

(a)

Problem 4.5

(a) The phase-modulated wave is

$$s(t) = A_c\cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)]$$

f_m $-f_m$

$$= A_c\cos[2\pi f_c t + \beta_p \cos(2\pi f_m t)]$$

$$= A_c\cos(2\pi f_c t)\cos[\beta_p\cos(2\pi f_m t)] - A_c\sin(2\pi f_c t)\sin[\beta_p\cos(2\pi f_m t)]$$

If $\beta_p \le 0.5$, then

$$\cos[\beta_p\cos(2\pi f_m t)] \approx 1$$

$$\sin[\beta_p\cos(2\pi f_m t)] \approx \beta_p\cos(2\pi f_m t)$$

Hence, we may rewrite Eq. (1) as

$$s(t) \approx A_c\cos(2\pi f_c t) - \beta_p A_c\sin(2\pi f_c t)\cos(2\pi f_m t)$$

$$= A_c\cos(2\pi f_c t) - \frac{1}{2}\beta_p A_c \sin[2\pi(f_c+f_m)t]$$

$$-\frac{1}{2}\beta_p A_c \sin[2\pi(f_c-f_m)t]$$

The spectrum of $s(t)$ is therefore

$$S(f) \approx \frac{1}{2}A_c[\delta(f-f_c)+\delta(f+f_c)]$$

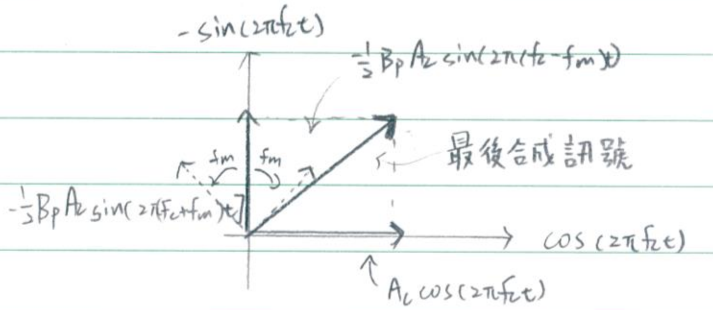
$$-\frac{1}{4j}\beta_p A_c[\delta(f-f_c-f_m)-\delta(f+f_c+f_m)]$$

$$-\frac{1}{4j}\beta_p A_c[\delta(f-f_c+f_m)-\delta(f+f_c-f_m)]$$

2.(b) phasor diagram

PM: $s(t) \approx A_c \cos(2\pi f_c t) - B_p A_c \sin(2\pi f_c t) \cos(2\pi f_m t)$

$= A_c \cos(2\pi f_c t) - \frac{1}{2} B_p A_c \sin[2\pi(f_c+f_m)t] - \frac{1}{2} B_p A_c \sin(2\pi(f_c-f_m)t)$



NBFM, $s(t) = A_c \cos(2\pi f_c t) - A_c \left\{ 2\pi k_f \int_0^t m(\tau) d\tau \right\} \sin 2\pi f_c t$

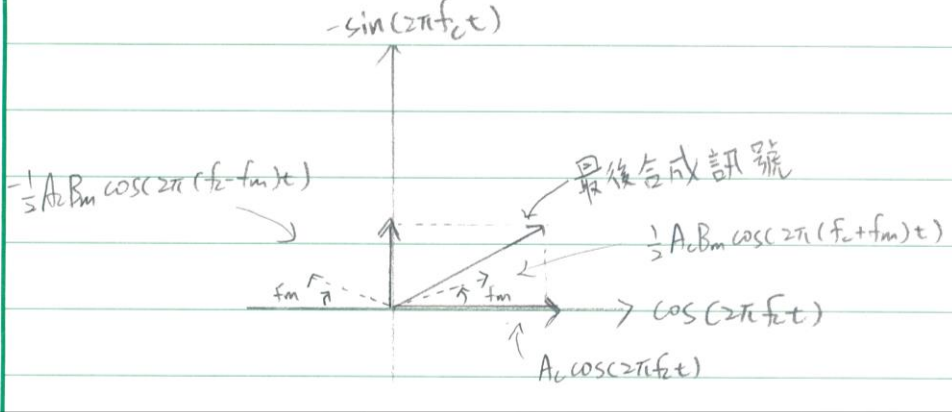
$= A_c \cos(2\pi f_c t) - A_c \left\{ 2\pi k_f \int_0^t A_m \cos(2\pi f_m \tau) d\tau \right\} \sin 2\pi f_c t$

$= A_c \cos(2\pi f_c t) - A_c A_m \frac{2\pi k_f}{2\pi f_m} \sin(2\pi f_m \tau) \Big|_0^t \sin(2\pi f_c t)$

$= A_c \cos(2\pi f_c t) - A_c B_m \sin(2\pi f_m t) \sin(2\pi f_c t)$

$= A_c \cos(2\pi f_c t) - A_c B_m \left(\frac{-\cos(2\pi(f_m+f_c)t) + \cos(2\pi(f_c-f_m)t)}{2} \right)$

$= A_c \cos(2\pi f_c t) + \frac{1}{2} A_c B_m \cos(2\pi(f_m+f_c)t) - \frac{1}{2} A_c B_m \cos(2\pi(f_c-f_m)t)$



$\cos(a+b)$
$= \cos a \cos b - \sin a \sin b$.

$\sin(\alpha+\beta)$
$= \sin\alpha \cos\beta + \cos\alpha \sin\beta$

Solution 3.

$$s(t) = A_c \cos(\theta(t))$$
$$\theta(t) = 2\pi f_c t + k_p m(t)$$

Let $\beta < 0.3$ for $m(t) = \cos(2\pi f_m t)$.

$$\therefore s(t) = A_c \cos(2\pi f_c t + \beta m(t))$$
$$= A_c[\cos(2\pi f_c t)\cos(\beta \cos(2\pi f_m t)) - \sin(2\pi f_c t)\sin(\beta \cos(2\pi f_m t))]$$

for small β:

$$\cos(\beta \cos(2\pi f_m t)) \approx 1$$
$$\sin(\beta \cos(2\pi f_m t)) \approx \beta \cos(2\pi f_m t)$$

$$\therefore s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\cos(2\pi f_m t)$$
$$= A_c \cos(2\pi f_c t) - \beta \frac{A_c}{2}[\sin(2\pi(f_c + f_m)t) + \sin(2\pi(f_c - f_m)t)]$$