Peak-to-Average Power Ratio (PAPR)

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Multi-Carrier Systems

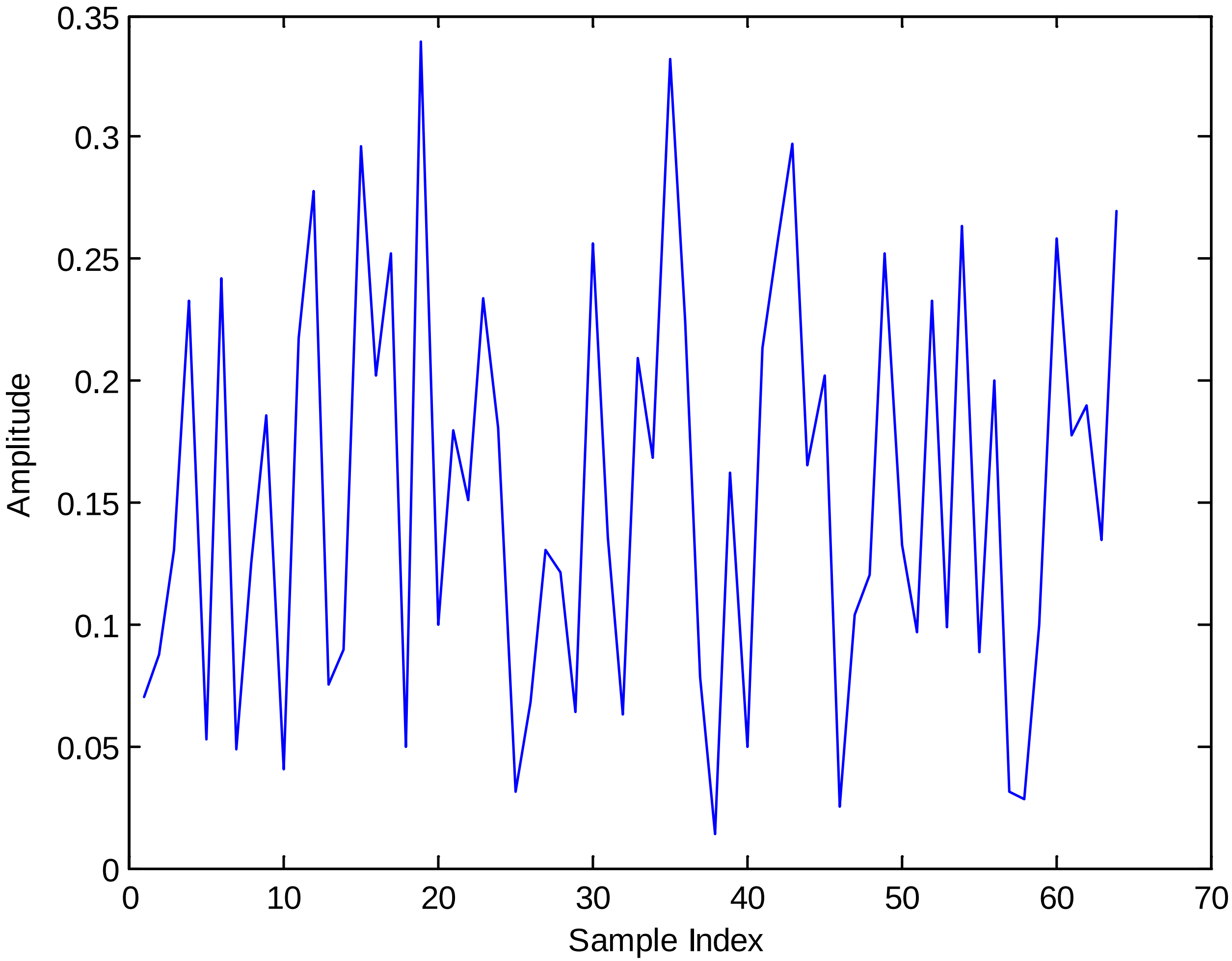
- The complex baseband representation of a multicarrier signal consisting of N subcarriers is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi n \Delta f t}, 0 \le t \le T$$

where Δf is the subcarrier spacing.

- In OFDM systems, the subcarriers are chosen to be orthogonal (i.e., $\Delta f = 1/T$)

An example of the time-domain signals with 64 subcarriers



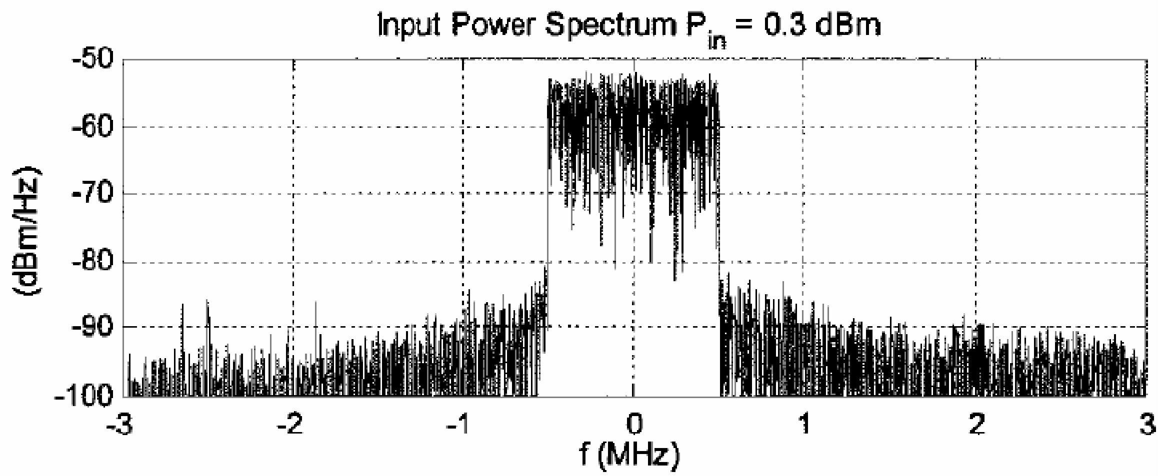
The effect of high PAPR

- Due to the large number of sub-carriers in typical OFDM systems, the amplitude of the transmitted signal has a large dynamic range, leading to in-band distortion and out-of-band radiation when the signal is passed through the nonlinear region of power amplifier.

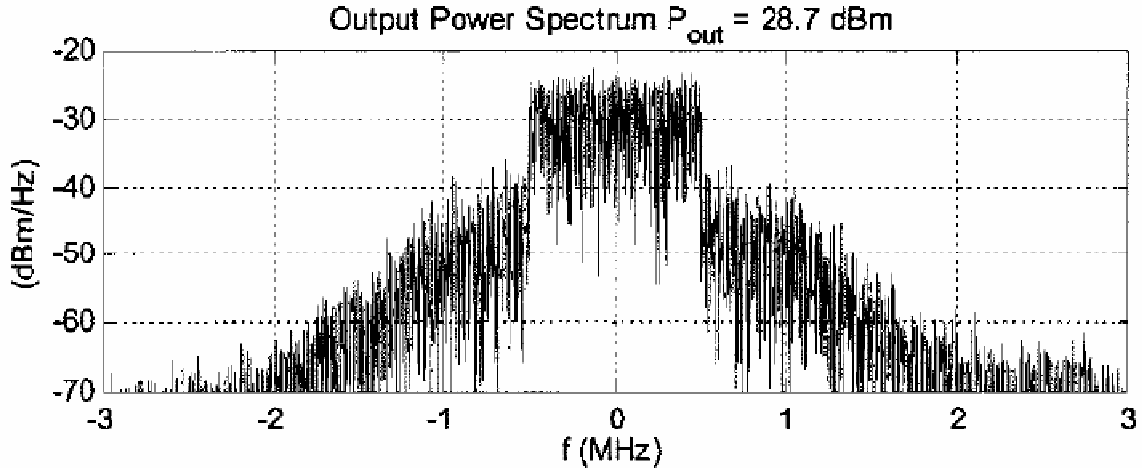
- The PAPR of the transmit signal is defined as

$$PAPR = \frac{\max_{0 \le t \le T} \left| x(t) \right|^2}{1/T \cdot \int_0^T \left| x(t) \right|^2 dt}$$

Bandwidth regrowth



(a)



PAPR in discrete-time case

- If we sample $x(t)$ by a sampling rate of $1/T_s$ (the sampling period $T_s = T/N$), we may miss some signal peaks and get optimistic results for the PAPR.

- For better approximating the true PAPR in the discrete-time case, we usually oversample $x(t)$ by a factor of L , i.e., the sampling rate is L/T_s .

- In general, an oversampling factor $L=4$ is sufficient to approximate the true PAPR.

PAPR in discrete-time case

- For an OFDM system with N sub-carriers, an oversampling rate of L can be achieved by inserting $(L-1)\cdot N$ zeros in the middle of the modulated symbol vector to form a $1\times LN$ data vector **X**, i.e.

$$\mathbf{X}=\left[X[0],\ X[1],\ \cdots,\ X[N/2-1],\ \underbrace{0,\ \cdots,\ 0}_{(L-1)\cdot N},\ X[N/2],\ X[N/2+1],\cdots,\ X[N-1]\right]$$

- Then, an LN-point IFFT process is performed to generate the oversampled time-domain signal vector **x**, where the nth element of **x** is given by

$$x[n]=\frac{1}{\sqrt{LN}}\sum_{k=0}^{LN-1}X[k]\cdot\exp\left(\frac{j2\pi nk}{LN}\right),$$

- The PAPR computed from the L-times oversampled time-domain signal samples is given by

$$PAPR=\frac{\max\limits_{0\le n\le LN-1}\left|x[n]\right|^2}{E\left[\left|x[n]\right|^2\right]}$$

The CCDF of the PAPR

- The cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. In the literature, the complementary CDF (CCDF) is commonly used instead of the CDF itself.

- The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold.

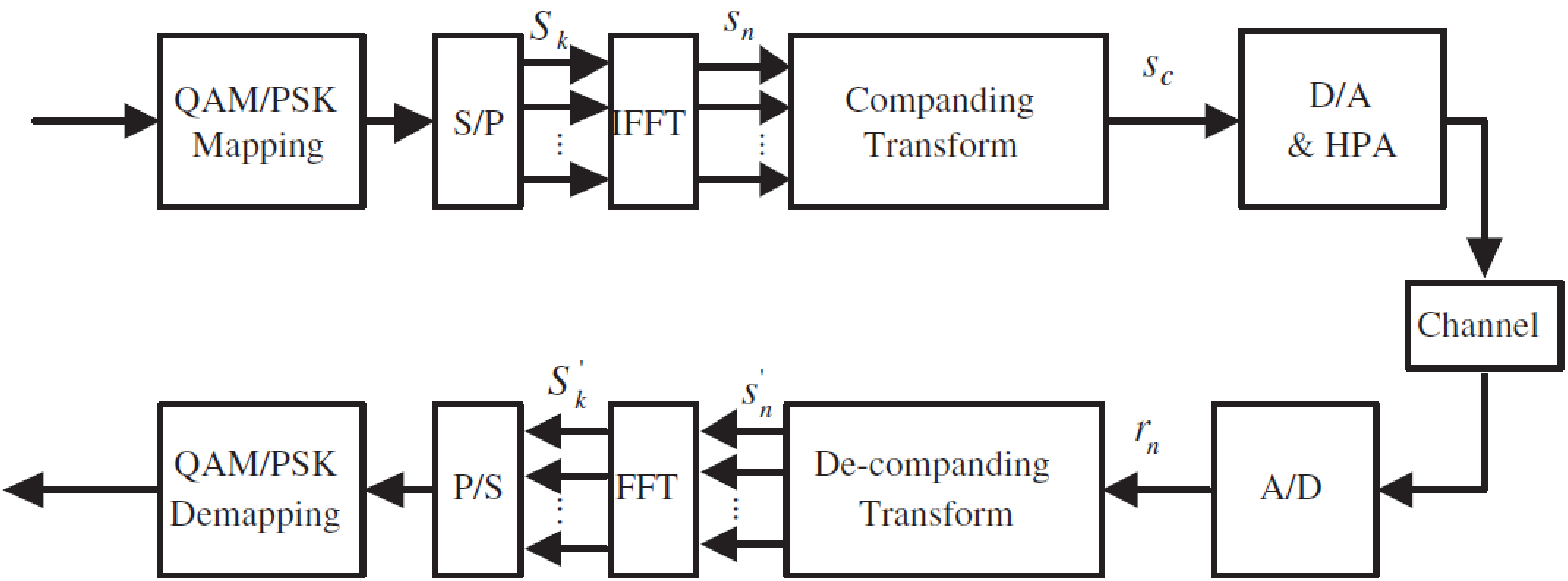
PAPR reduction methods

- Distortion
  - Clipping
  - Companding
  - Active Constellation Extension
- Distortionless
  - Selected Mapping (SLM)
  - Partial Transmit Sequence (PTS)

Clipping

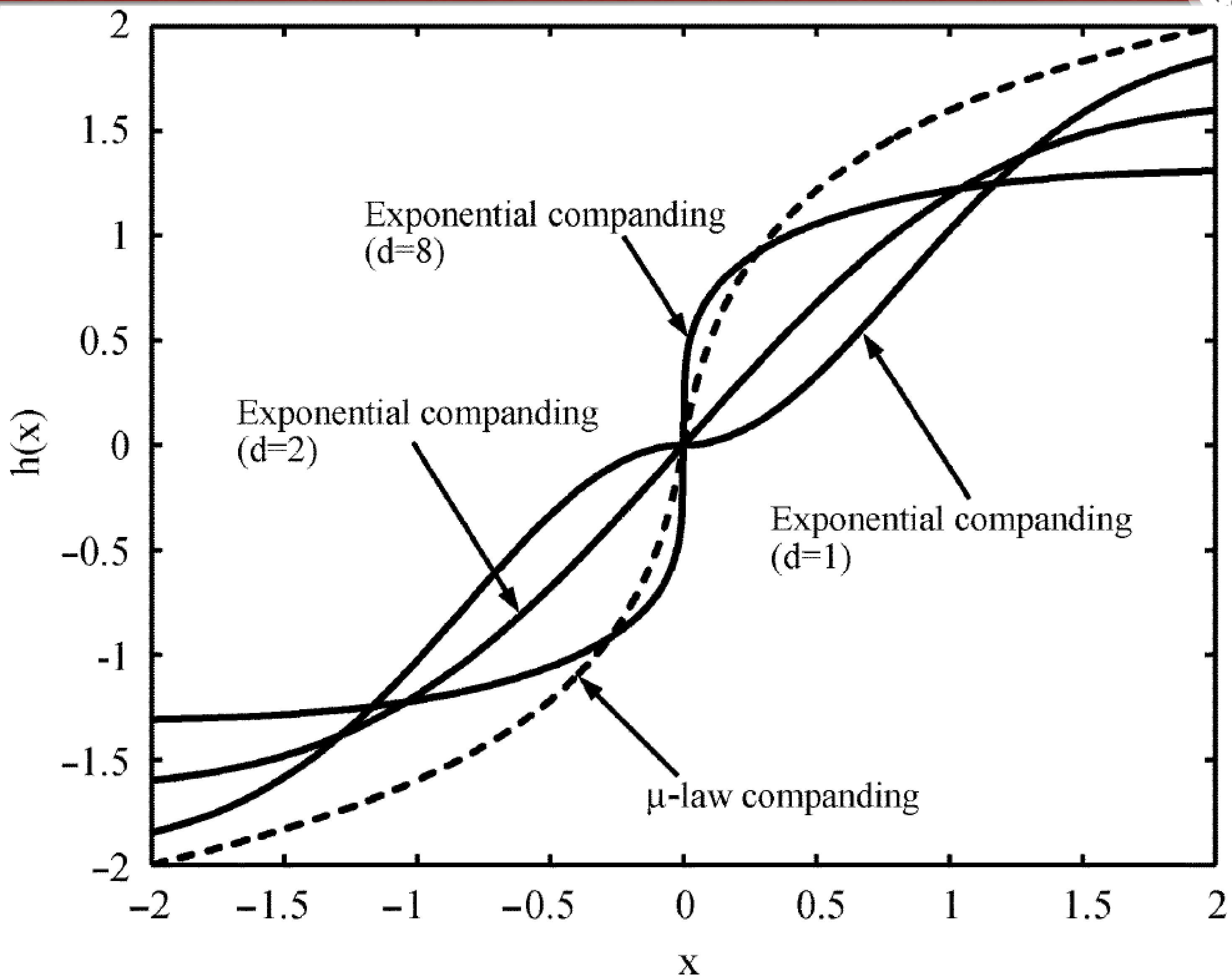
- The simplest way to reduce the PAPR.
- The peak amplitude becomes limited to a desired level.
- Clipping $$y_n = \begin{cases} x_n & ,|x_n| \le A \\ A \cdot \exp\{\arg(x_n)\} & ,|x_n| > A \end{cases}$$
- By distorting the OFDM signal amplitude, a kind of self-interference introduced that degrades the BER.
- Nonlinear distortion increases out-of-band radiation.

Companding



QAM/PSK Mapping
S/P
S_k
IFFT
s_n
Companding Transform
s_c
D/A & HPA
Channel
A/D
r_n
De-companding Transform
s'_n
FFT
S'_k
P/S
QAM/PSK Demapping

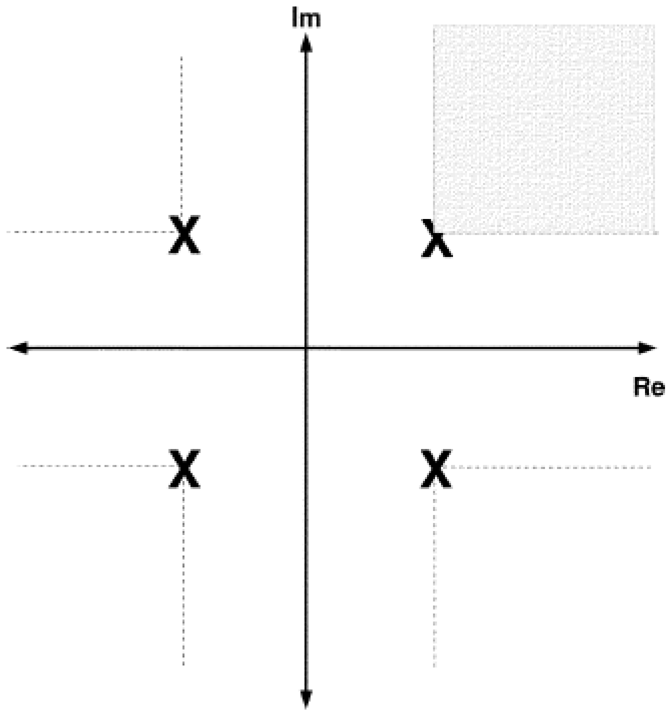
Companding



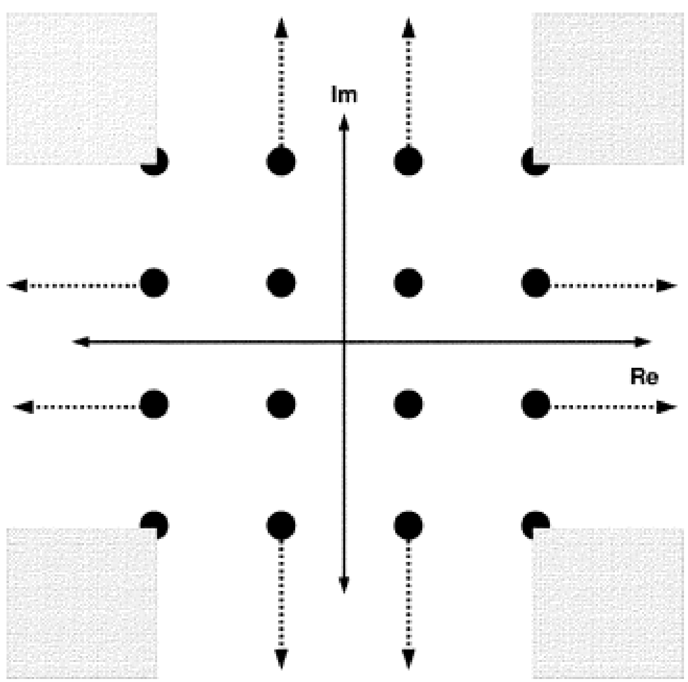
Exponential companding
(d=8)
Exponential companding
(d=2)
Exponential companding
(d=1)
μ-law companding
h(x)
x
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Active constellation extension

- In this technique, some of the outer signal constellation points in the data block are dynamically extended toward the outside of the original constellation such that the PAPR of the data block is reduced.

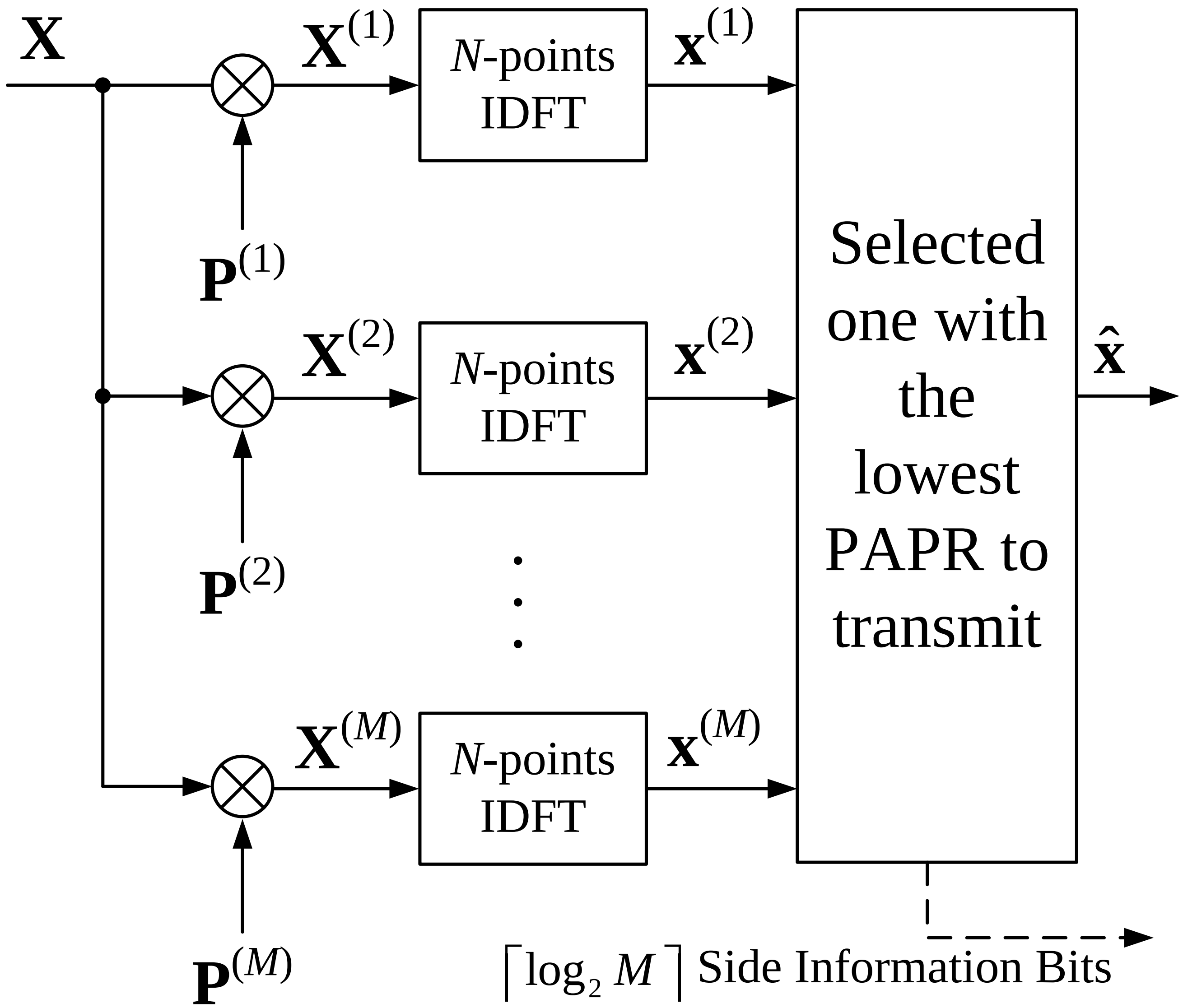


QPSK



16-QAM

Selected mapping (SLM)



Selected mapping (SLM)

- Consider an OFDM system with N sub-carries.
- Firstly, the $N\times1$ modulated symbol vector, $\mathbf{X}$, is sent into M branches. $\mathbf{P}^{(m)}$ is a phase rotation vector for the mth branch and each element of $\mathbf{P}^{(m)}$ belongs to $[0, 2\pi)$ for $m=1, 2, \ldots, M$.
- Following, performing element-wise multiplication between $\mathbf{X}$ and $\mathbf{P}^{(m)}$ obtains $\mathbf{X}^{(m)}$, which is the mth candidate signal in the frequency domain.
- To obtain the time-domain candidate signal $\mathbf{x}^{(m)}$, the N-points IDFT is performed to $\mathbf{X}^{(m)}$.
- After generating M candidate signals, the candidate signal with the lowest PAPR is selected to transmit.

PAPR reduction performance of SLM

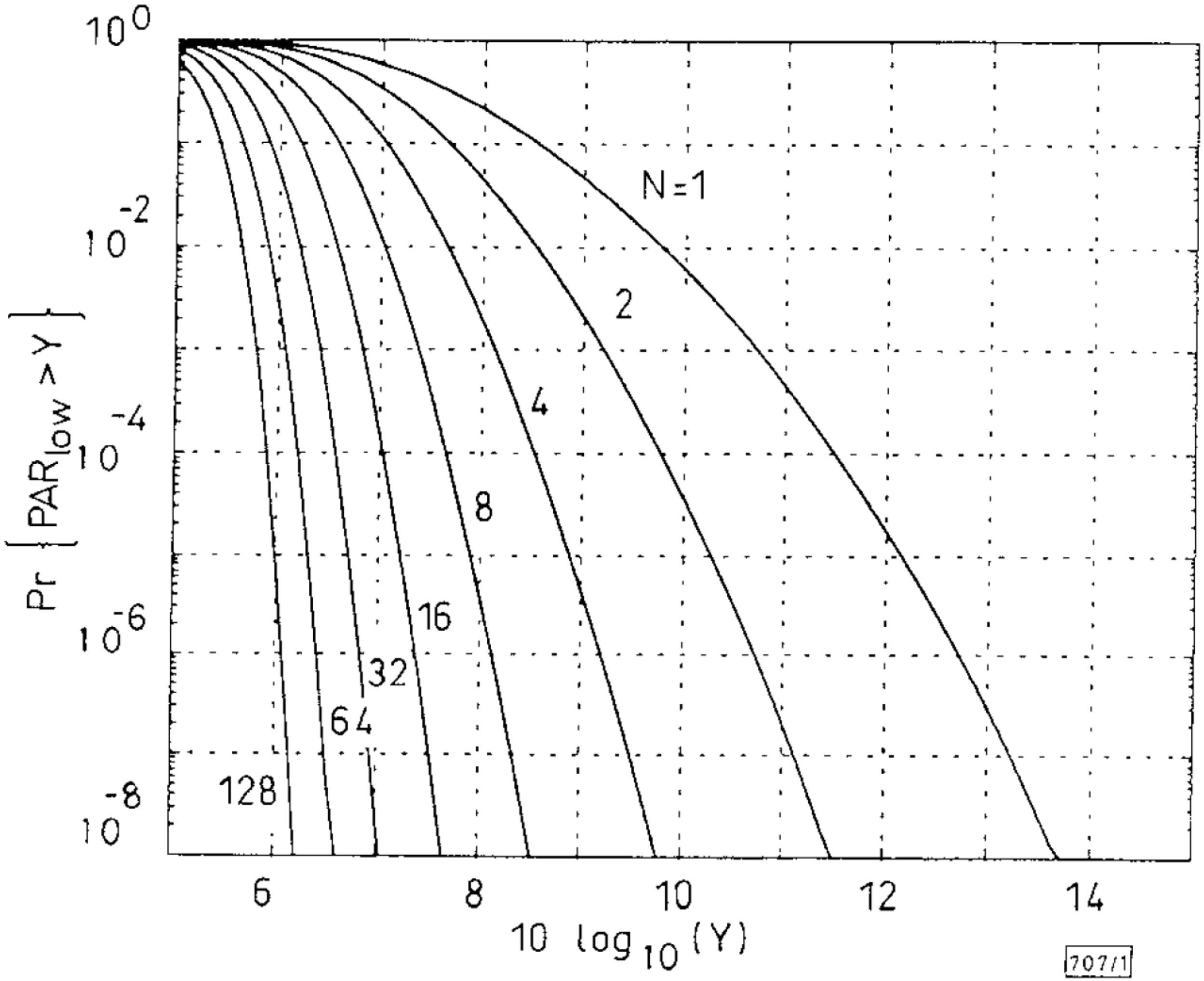
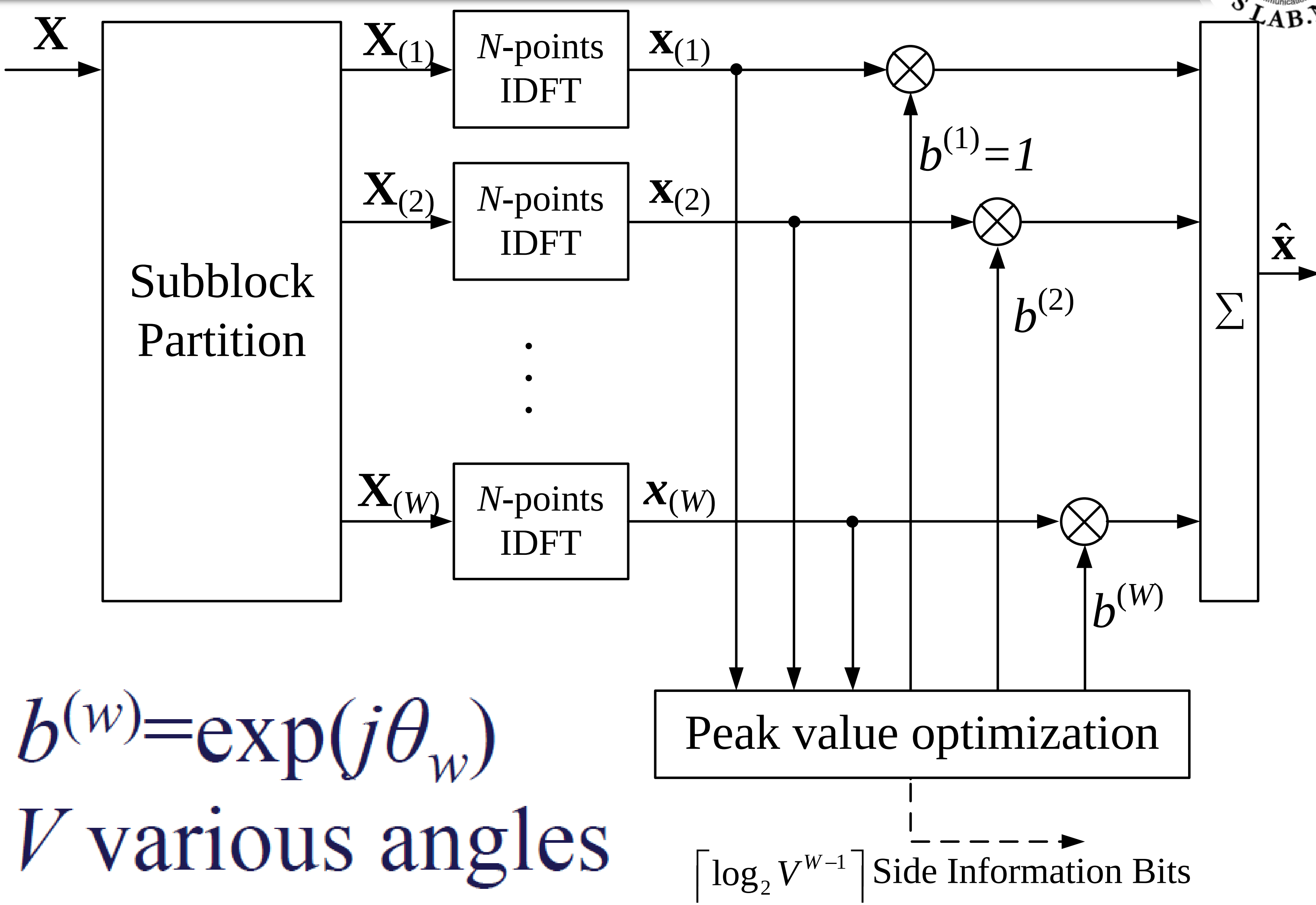


Fig. 1 *Complementary cumulative distribution function of PAR, if frame with lowest PAR is selected out of N statistically independent frames*

Partial transmit sequence (PTS)



$b^{(w)} = \exp(j\theta_w)$

V various angles

Partial transmit sequence (PTS)

- Firstly, N subcarriers are divided into W subblocks. That is, the $N\times1$ modulated symbol vector $\mathbf{X}$ is divided into W $N\times1$ vectors, $\mathbf{X}_{(1)}$, $\mathbf{X}_{(2)}$, … $\mathbf{X}_{(W)}$, and the nonzero elements of $\mathbf{X}_{(1)}$, $\mathbf{X}_{(2)}$, … $\mathbf{X}_{(W)}$, consists of $\mathbf{X}$.
- Performing IDFT to $\mathbf{X}_{(1)}$, $\mathbf{X}_{(2)}$, … $\mathbf{X}_{(W)}$, obtains time-domain vectors $\mathbf{x}_{(1)}$, $\mathbf{x}_{(2)}$, … $\mathbf{x}_{(W)}$.
- Sending these vectors to the peak value optimization block to find out the optimization phase rotation $b^{(1)}$, $b^{(2)}$, …, $b^{(W)}$, which cause the sum of $b^{(w)}\cdot\mathbf{x}_{(w)}$ with the lowest PAPR value. Note that $b^{(w)}=\exp(j\theta_w)$ and V various angles can be selected be θ_w for $w=1, 2,…, W$. Moreover, $b^{(1)}=1$.
- Therefore, it requires $\lceil \log_2 V^{W-1} \rceil$ side information bits.

PAPR reduction performance of PTS

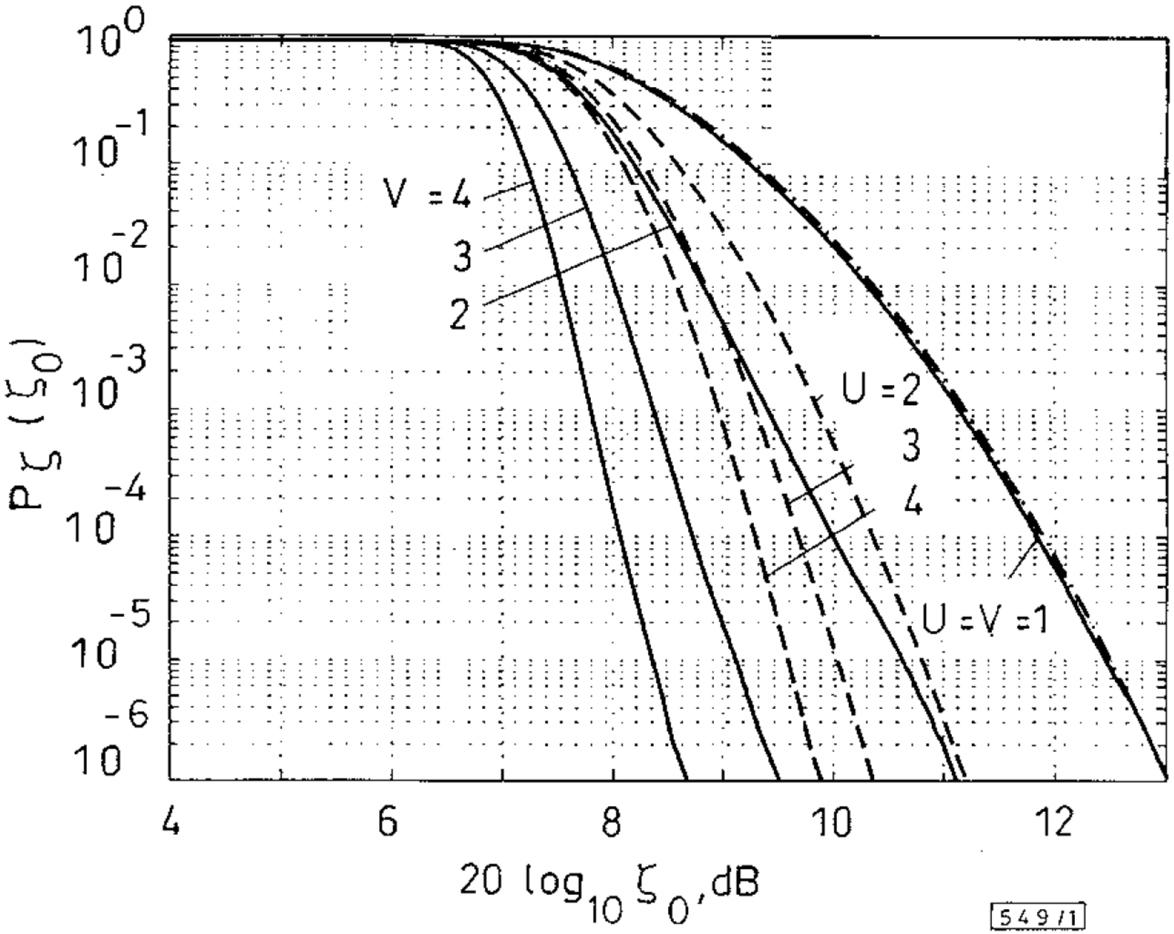


Fig. 1 *Probability $P_\zeta(\zeta_0)$ that OFDM symbol with $D = 512$ exceeds crest factor ζ_0 if PTS or SLM is used with $V, U \in \{1,2,3,4\}$*

——— PTS-OFDM

– – – – SLM-OFDM