Communication Systems

2019 Fall – Midterm

Thursday, November 14, 2019.

1. Closed book.
2. 13:10~16:00 (170 minutes).

$$\operatorname{rect}(t)=\begin{cases}1, & -\frac{1}{2}<t<\frac{1}{2}\\ 0, & |t|\geq\frac{1}{2}\end{cases}\quad \operatorname{sinc}(\lambda)\equiv\frac{\sin(\pi\lambda)}{\pi\lambda}\quad u(t)=\begin{cases}1, & t>0\\ \frac{1}{2}, & t=0\\ 0 & t<0\end{cases}\quad \operatorname{sgn}(t)=\begin{cases}+1, & t>0\\ 0, & t=0\\ -1, & t<0\end{cases}$$

Problem 1 [10]: Fourier Transform

(1). [2] Given $\exp(-at)u(t)\rightleftharpoons\frac{1}{a+j2\pi f}$, $a>0$, please find the Fourier transform of a double exponential pulse $\exp(-a|t|)$, $a>0$.

(2). [2] Please find the Fourier transform of $\operatorname{sgn}(t)$.

(3). [2] Please find the Fourier transform of a unit step function $u(t)$.

(4). [2] Please find the Fourier transform of $g(t)=\exp(-t)\sin(2\pi f_c t)u(t)$.

(5). [2] Please find the energy of the sinc pulse: $E=A^2\int_{-\infty}^{\infty}\operatorname{sinc}^2(2Wt)\,dt$.

Problem 2. [10] Prove the following properties.

(1). [2] $g_1(t)*g_2(t)=g_2(t)*g_1(t)$.

(2). [2] $g_1(t)*\left[g_2(t)*g_3(t)\right]=\left[g_1(t)*g_2(t)\right]*g_3(t)$.

(3). [2] $g_1(t)*\left[g_2(t)+g_3(t)\right]=g_1(t)*g_2(t)+g_1(t)*g_3(t)$.

(4). [2] $\frac{d}{dt}\left[g_1(t)*g_2(t)\right]=\frac{d}{dt}\left[g_1(t)\right]*g_2(t)$.

(5). [2] $\int_{-\infty}^{t}\left[g_1(\tau)*g_2(\tau)\right]d\tau=\left[\int_{-\infty}^{t}g_1(\tau)\,d\tau\right]*g_2(t)$.

Problem 3. [10] The Fourier transform of a signal $g(t)$ is denoted by $G(f)$. Prove the following properties of the Fourier transform:

(1). [2] If a real signal $g(t)$ is an even function of time t, the Fourier transform $G(f)$ is purely real. If a real signal $g(t)$ is an odd function of time t, the Fourier transform $G(f)$ is purely imaginary.

(2). [2] $t^n g(t)\Leftrightarrow\left(\frac{j}{2\pi}\right)^n G^{(n)}(f)$, where $G^{(n)}(f)$ is the nth derivative of $G(f)$ with respect to f.

(3). [2] $\int_{-\infty}^{\infty}t^n g(t)\,dt\Leftrightarrow\left(\frac{j}{2\pi}\right)^n G^{(n)}(0)$.

(4). [2] $\int_{-\infty}^{\infty}g_1(t)g_2^*(t)dt\Leftrightarrow\int_{-\infty}^{\infty}G_1(f)G_2^*(f)\,df$.

(5). [2] $g_1(t)g_2^*(t)\Leftrightarrow\int_{-\infty}^{\infty}G_1(\lambda)G_2^*(\lambda-f)\,d\lambda$.

Problem 4. [10] The signal $x(t) = 30\text{sinc}(60t)$ is applied to the nonlinear modulator system shown in the following figure with a carrier frequency of $f_c = 1$ kHz. Assume that $v_{out} = a_1 v_{in} + a_2 v_{in}^3$.

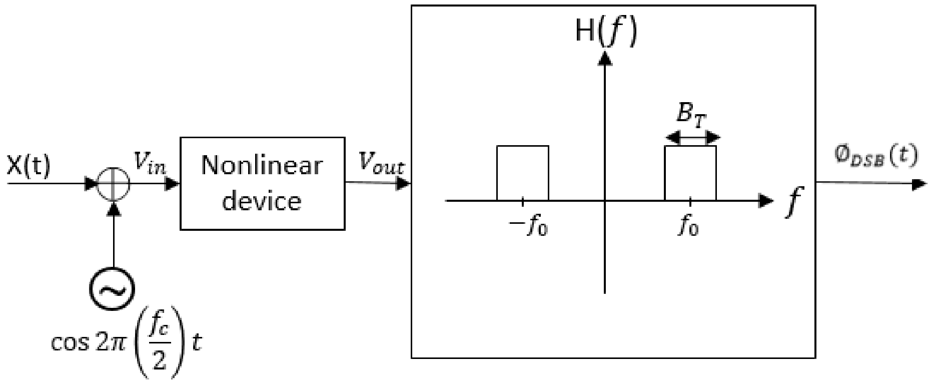
(1). [2] Sketch the spectrum of $x(t)$.

(2). [2] Determine the output signal $v_{out}(t)$.

(3). [2] Give the conditions on the required center frequency f_0 and the bandwidth of the bandpass filter in relation to f_c and the bandwidth of the signal $x(t)$ such that this system will produce a standard DSB signal.

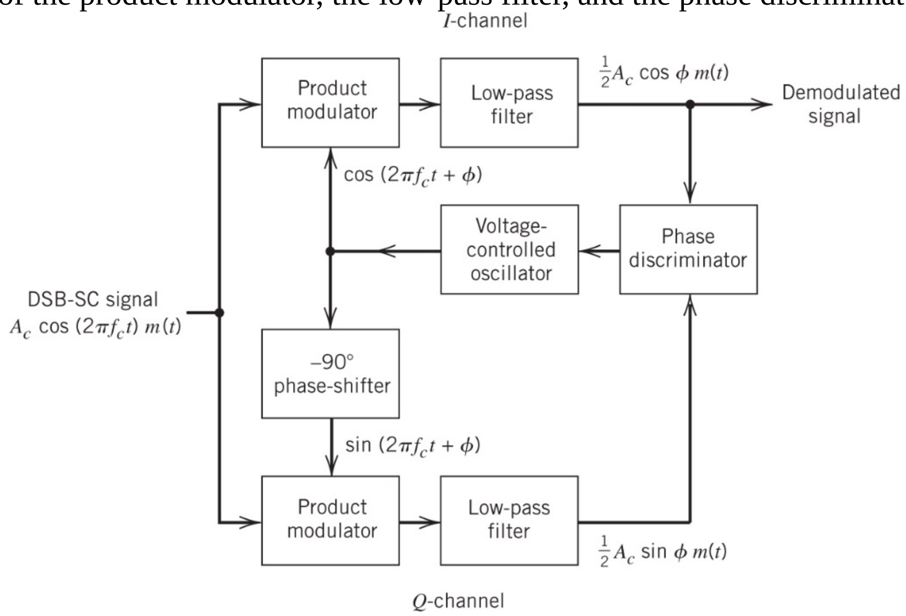
(4). [2] Determine the output DSB signal $\phi_{DSB}(t)$.

(5). [2] Determine values of a_1 and a_2 such that the maximum value of the amplitude of the output signal $\phi_{DSB}(t)$ is 10.



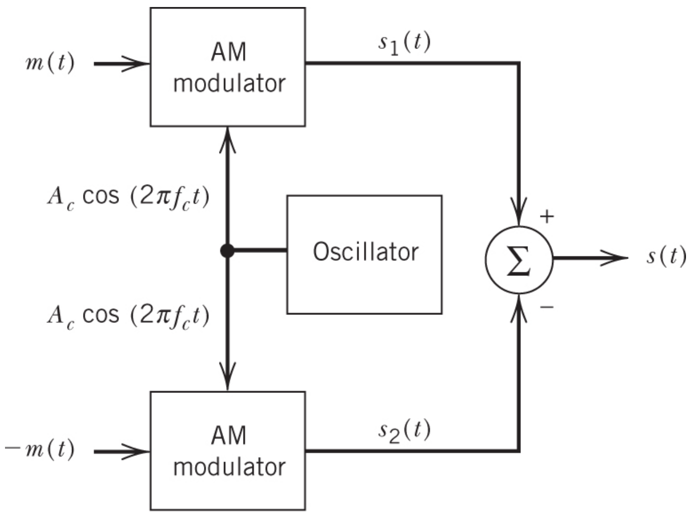
Problem 5. [10] Costas Receiver

Describe the operations of a Costas Receiver as shown in the following figure. (Please derive the output signals of the product modulator, the low-pass filter, and the phase discriminator.)



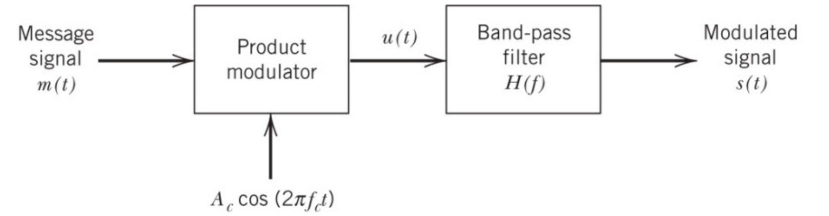
Problem 6. [10] Show that an AM signal can be demodulated using coherent demodulation.

Problem 7. [10] The following figure shows the circuit diagram of a ***balance modulator.*** The input applied to the top AM modulator is $m(t)$, whereas that applied to the lower AM modulator is $-m(t)$; these two modulators have the same amplitude sensitivity. Show that the output $s(t)$ of the balanced modulator consists of a DSB-SC modulated signal.

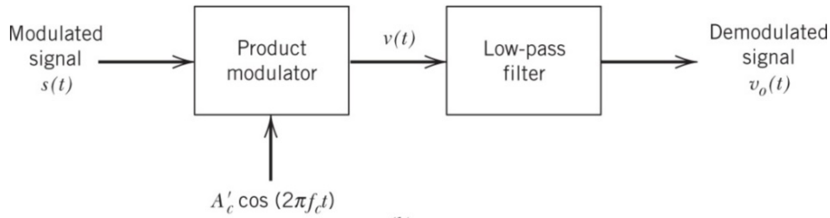


Problem 8. [10] VSB Modulation

Consider the models shown in the following figure for generating the VSB signal and for coherently detecting it. Please derive the condition of the filter $H(f)$ to permit accurate recovery of the message signal.



(a)

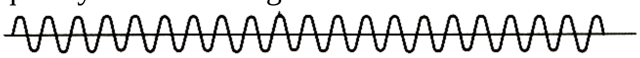


(b)

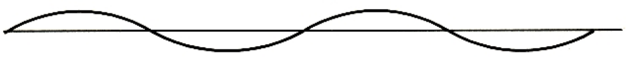
Problem 9. [10] Consider the following figure where (a) is the carrier wave and (b) is the sinusoidal modulating signal. Please answer the following questions and justify your answers.

(1). [2] Which is the phase modulated signal?

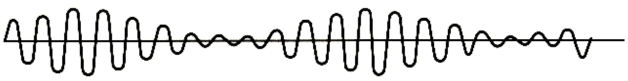
(2). [2] Which is the frequency modulated signal?



(a)

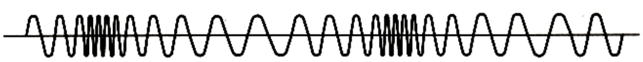


(b)



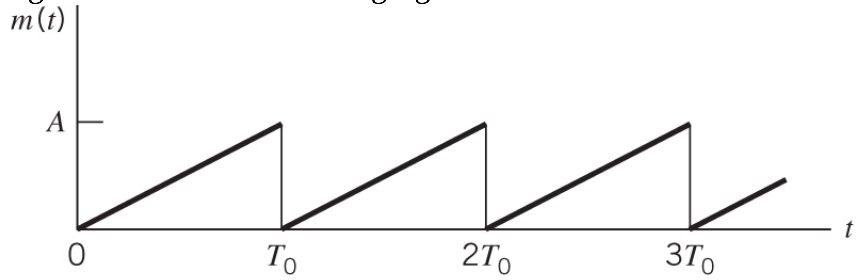
(c)

(d)



(e)

For the message signal shown in the following figure



(3). [3] Please draw a diagram to illustrate the phase modulated wave.

(4). [3] Please draw a diagram to illustrate the frequency modulated wave.

Problem 10. [10] The sinusoidal modulating wave $m(t) = A_m \cos(2\pi f_m t)$ is applied to a phase modulator with phase sensitivity k_p. The unmodulated carrier wave has frequency f_c and amplitude A_c.

(1). [5] Determine the spectrum of the resulting phase-modulated signal, assuming that the maximum phase deviation $\beta_p = k_p A_m$ does not exceed 0.3 radians.

(2). [5] Construct a phasor diagram for this modulated signal, and compare it with that of the corresponding narrow-band FM signal.

Properties of the Fourier Transform

Property	Mathematical Description
Linearity	$ag_1(t)+bg_2(t) \rightleftharpoons aG_1(f)+bG_2(f)$, where a and b are constants.
Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$, where a is a constant.
Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$.
Time shifting	$g(t-t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$.
Frequency shifting	$\exp(j2\pi f_c t)\,g(t) \rightleftharpoons G(f-f_c)$.
Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$.
Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$.
Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$.
Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$.
Multiplication in the time domain	$g_1(t)\,g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G(f-\lambda)\,d\lambda$.
Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$.
Rayleigh's energy theorem	$\int_{-\infty}^{\infty} \lvert g(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2 df$.

$$2\sin\alpha\cdot\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cdot\sin\beta=\sin(\alpha+\beta)-\sin(\alpha-\beta)$$
$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$
$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$
$$\left[\sin\left(\frac{A}{2}\right)\right]^2=\frac{1-\cos A}{2}$$

$$2\sin\alpha\cdot\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\cos\alpha\cdot\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$
$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$
$$\left[\cos\left(\frac{A}{2}\right)\right]^2=\frac{1+\cos A}{2}$$

$\mathrm{rect}\left(\frac{t}{T}\right) \Leftrightarrow T\,\mathrm{sinc}(fT)$	$\mathrm{sinc}(2Wt) \Leftrightarrow \frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(j2\pi f_c t) \Leftrightarrow \delta(f-f_c)$	$\mathrm{sgn}(t) \Leftrightarrow \frac{1}{j\pi f}$
$\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]$	$\sin(2\pi f_c t) \Leftrightarrow \frac{1}{2j}\left[\delta(f-f_c)-\delta(f+f_c)\right]$