Introduction to OFDM Systems

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Outline

- OFDM Overview
- OFDM System Model
- Orthogonality
- Multi-carrier Equivalent Implementation by Using IDFT (IFFT)
- Cyclic Prefix (CP)

OFDM Overview

OFDM Overview

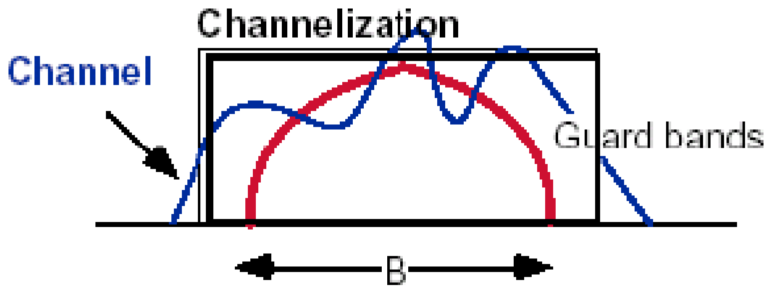
- OFDM
  - **O**rthogonal **F**requency **D**ivision **M**ultiplexing

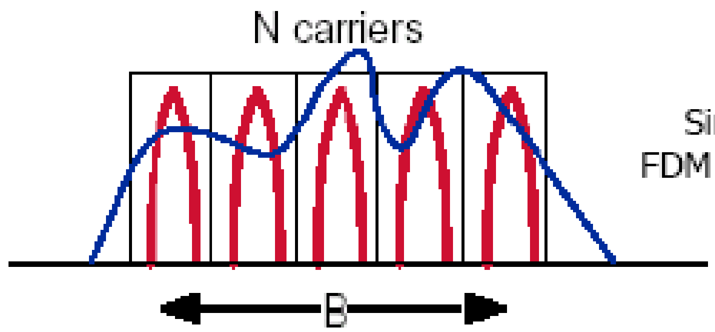
- Frequency Division Multiplexing (FDM) or multi-tone systems have been employed in military applications since the 196Os.

- OFDM employs multiple carriers overlapping in the frequency domain.

OFDM Overview

- Single carrier (SC) vs. multi-carrier (MC)





- Single carrier : data are transmitted over only one carrier
- Selective fading

- Multi-carrier : data are shared among several carriers and simultaneously transmitted
- Flat fading per subcarrier

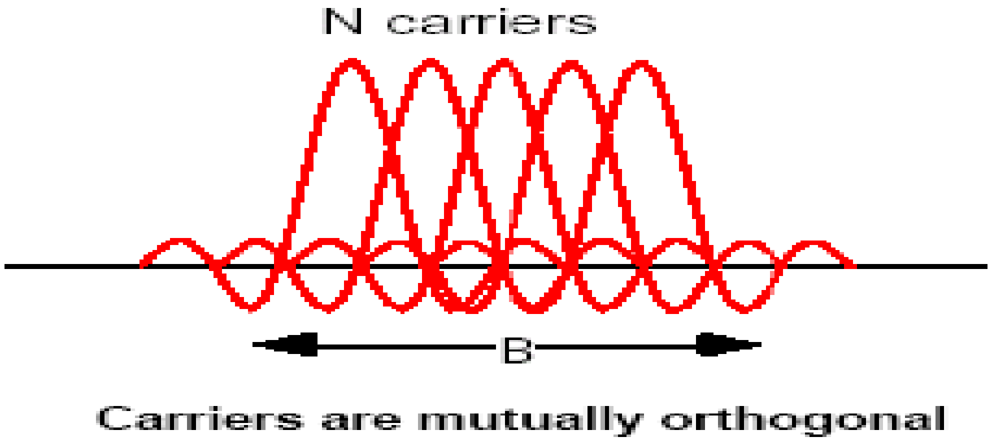
OFDM Overview

- The basic principle of OFDM is to split a high-rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of sub-carriers.

- It eliminates or alleviates the problem of multi-path channel fading effect, low spectrum efficiency, and frequency selective fading.

OFDM Overview

- OFDM modulation



  - Features
    - No intercarrier guard bands
    - Overlapping of bands
    - Spectral efficiency
    - Easy implementation by IFFTs
    - Very sensitive to synchronization

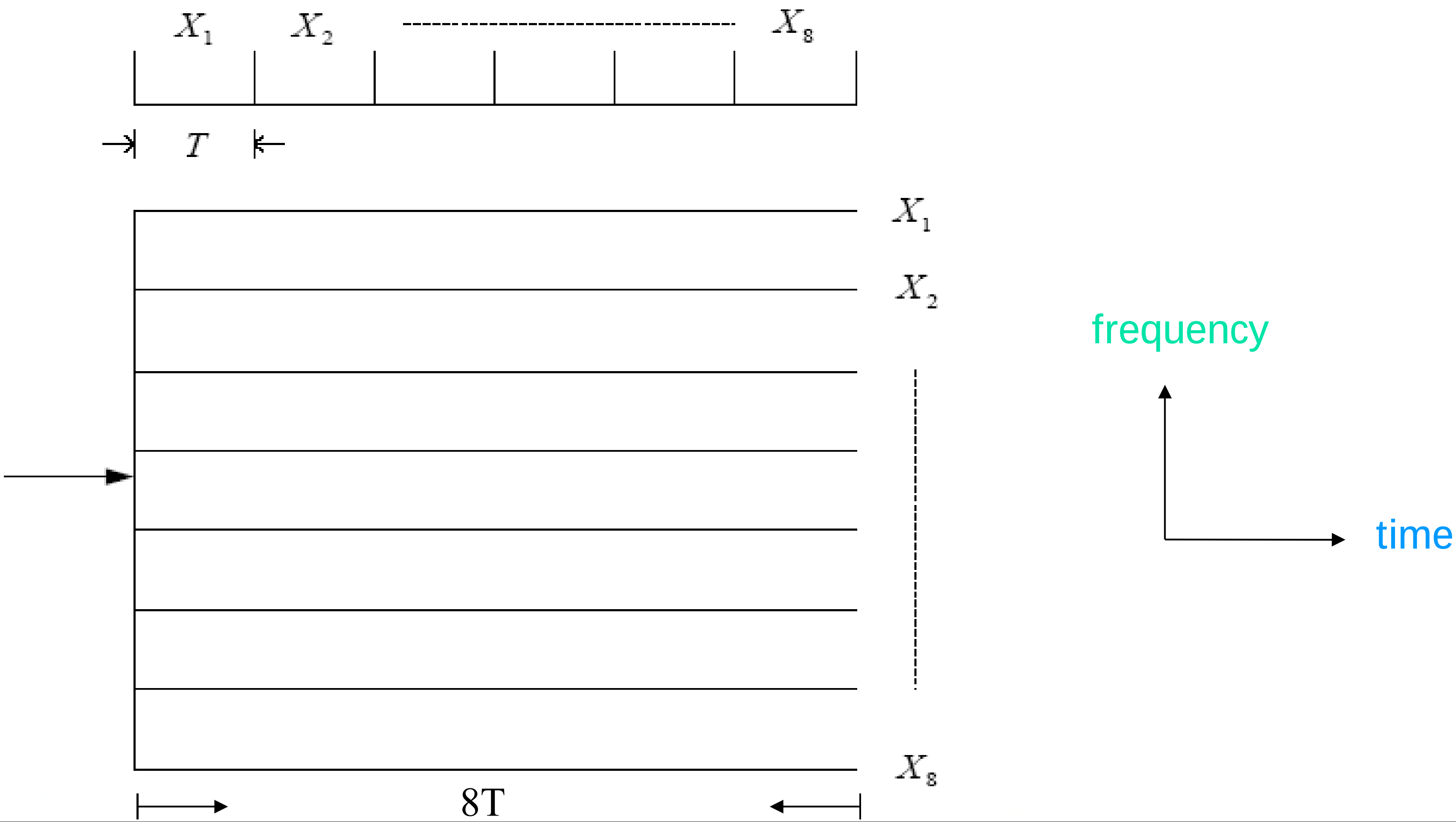
Applications of OFDM Technology

- Broadband Wired Access: Asymmetric Digital Subscriber Loop (ADSL), Digital Multi-tone (DMT).
- Wireless LANs (IEEE 802.11a/g, IEEE 802.11n, HIPERLAN-2)
- Digital Broadcasting (DAB, DVB-T, DVB-H)
- WiMAX (IEEE 802.16 Series), 3GPP Long Term Evolution (3GPP LTE), 4G.
- Wireless Personal Area Network (WPAN): IEEE 802.15a/MBOA
- Power Line

OFDM System Model

OFDM System Model

- Multi-carrier Block Transmission



OFDM System Model

- OFDM: A block modulation scheme that transmits a block N source symbols in parallel by using subcarriers
  - Sub-carriers are orthogonal in time, but overlapped in frequency.
  - Frequency spacing: $\Delta f = \frac{1}{T_{FFT}}$

$$\int_0^{T_{FFT}} \cos(2\pi f_1 t)\cos(2\pi (f_1 + \Delta f)t)dt = 0$$

OFDM System Model

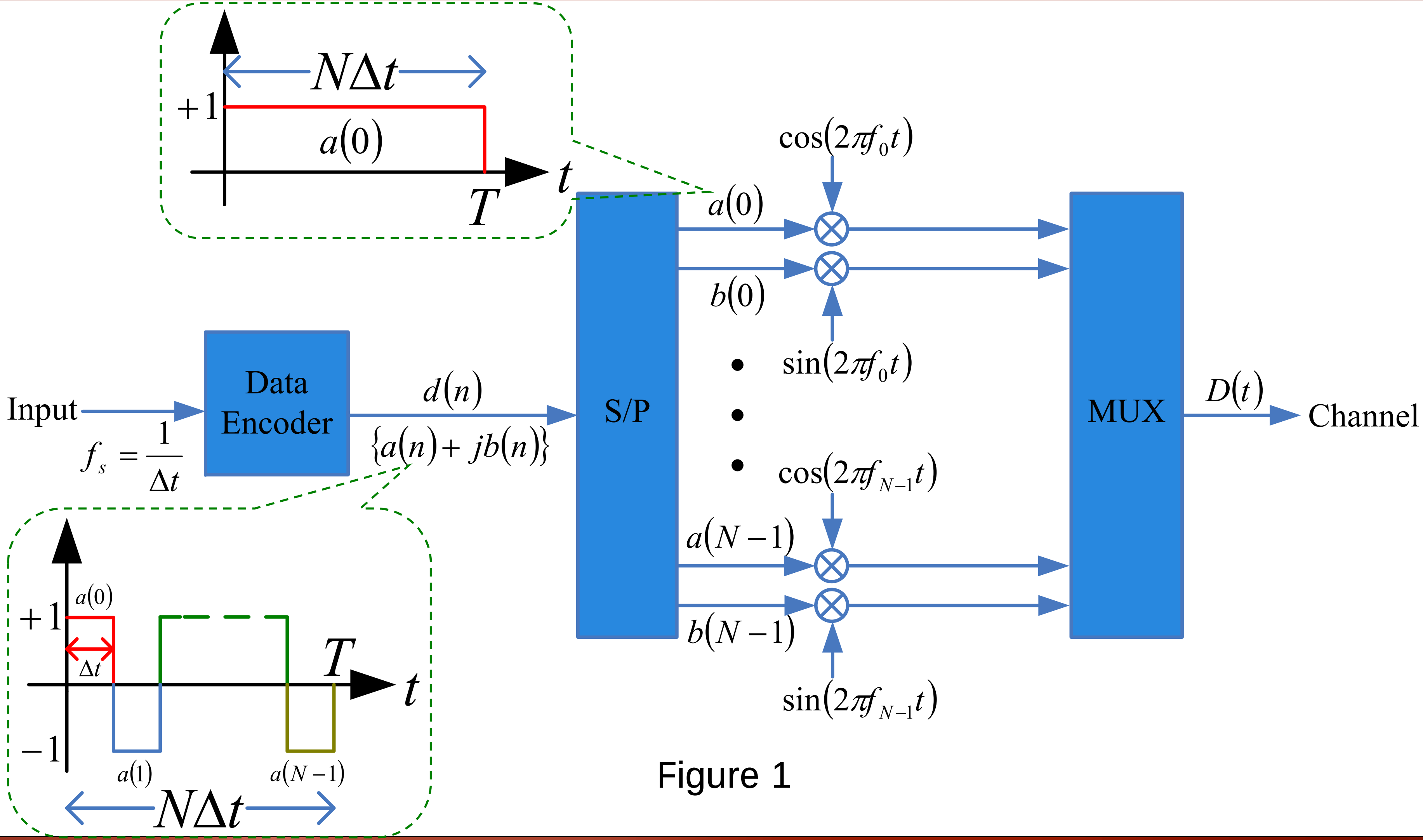


Figure 1

OFDM System Model

- An OFDM system transmitter shown in Figure 1.
- The transmitted waveform $D(t)$ can be expressed as

$$D(t) = \sum_{n=0}^{N-1} \{a(n)\cos(2\pi f_n t) + b(n)\sin(2\pi f_n t)\} \qquad (1)$$

where $f_n = f_0 + n\Delta f$ and $\Delta f = \dfrac{1}{N\Delta t}$

- Using a two-dimensional digital modulation format, the data symbols $d(n)$ can be represented as $a(n) + jb(n)$
  - $a(n)$: in-phase component
  - $b(n)$: quadrature component

OFDM System Model

- The serial data elements spaced by Δt are grouped and used to modulate N carriers. Thus they are frequency division multiplexed.

- The signaling interval is then increased to $N\Delta t$, which makes the system less susceptible to channel delay spread impairments.

Small-scale fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol preiod

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol preiod

Orthogonality

Orthogonality

- Consider a set of transmitted carriers as follows:

$$\psi_n(t) = e^{j2\pi\left(f_0 + \frac{n}{N\Delta t}\right)t} \quad \text{for} \;\; n = 0, 1, \ldots, N-1 \qquad (2)$$

$$\int_a^b \psi_p(t)\psi_q^*(t)dt = \begin{cases} (b-a) & \text{for} \;\; p = q \\ 0 & \text{for} \;\; p \neq q \;\; \text{and} \;\; (b-a) = N\Delta t \end{cases}$$

Orthogonality

$$\int_a^b \psi_p(t)\psi_q^*(t)dt = \int_a^b e^{j2\pi(p-q)\frac{t}{N\Delta t}}dt$$

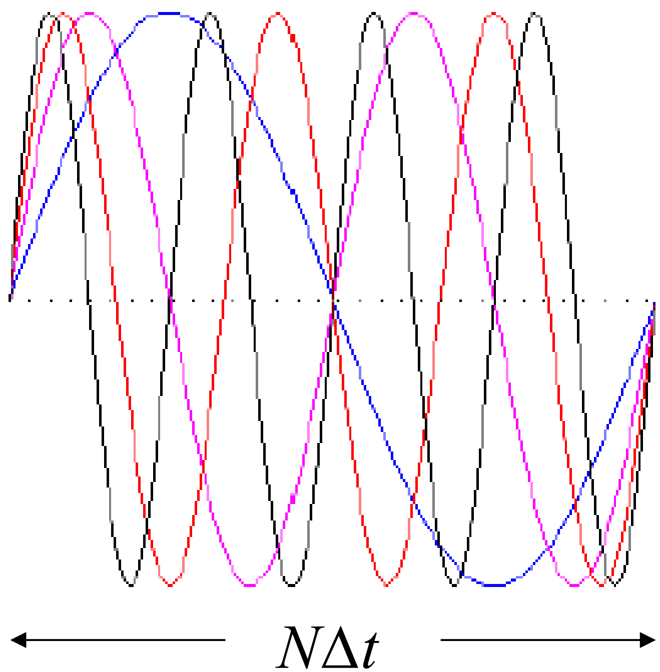
$$= \frac{e^{j2\pi(p-q)\frac{b}{N\Delta t}} - e^{j2\pi(p-q)\frac{a}{N\Delta t}}}{j2\pi(p-q)/N\Delta t}$$

$$= \frac{e^{j2\pi(p-q)\frac{b}{N\Delta t}}\left(1 - e^{j2\pi(p-q)\frac{1}{N\Delta t}(a-b)}\right)}{j2\pi(p-q)/N\Delta t}$$

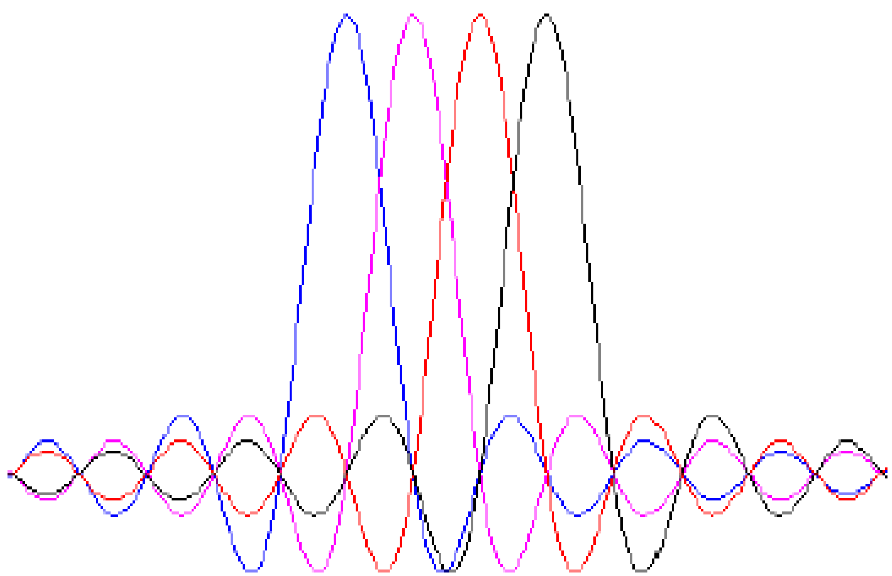
$$= 0\,,\ \text{for } p \neq q \ \text{ and } \ (b-a) = N\Delta t$$

Orthogonality

Time domain | **Frequency domain**



Example of four subcarriers within one OFDM symbol



Spectra of individual subcarriers

Mathematical Expression of OFDM Signal

- From above, we know that $\{\psi_n(t)\}$ is the orthogonal signal set. An OFDM signal based on this orthogonal signal set can be written as:

$$x(t) = \mathrm{Re}\left\{ \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} d_{k,n} \psi_n(t-kT) \right\} \qquad (3)$$

where $\psi_n(t) = e^{j2\pi f_n t}$ for $n = 0, 1, 2, ..., N-1 \quad 0 \le t \le T$

$$f_n = f_0 + \frac{n}{T}, \quad T = N\Delta t$$

$$d_{k,n} = a_{k,n} + jb_{k,n}$$

Mathematical Expression of OFDM Signal

- T : OFDM symbol duration
- $d_{k,,n}$: transmitted data on the n-th carrier of the k-th symbol

$$x(t) = \mathrm{Re}\left\{ \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} C_{k,n} \psi_n (t - kT) \right\}$$

$$= \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} \left\{ a_{k,n} \cos\left(2\pi f_n (t - kT)\right) - b_{k,n} \sin\left(2\pi f_n (t - kT)\right) \right\} \quad (4)$$

- If there is only one OFDM symbol (i.e. $k = 0$), it can be simplified as:

$$x(t) = \sum_{n=0}^{N-1} \{ a_n \cos(2\pi f_n t) - b_n \sin(2\pi f_n t) \} \quad (5)$$

Multi-carrier Equivalent Implementation by Using IDFT (IFFT)

Multi-carrier Equivalent Implementation by using IDFT

- According to the structure of Tx, it must use N oscillators. That increases the hardware complexity.

- The equivalent method is using IDFT (IFFT).

Multi-carrier Equivalent Implementation by using IDFT

- In general, each carrier can be expressed as:

$$S_c(t) = A_c(t) e^{j\left(2\pi f_c t + \phi_c(t)\right)} \quad (6)$$

- We assume that there are N carriers in the OFDM signal. Then the total complex signal $S_s(t)$ can be represented by:

$$S_s(t) = \frac{1}{N} \sum_{n=0}^{N-1} A_n(t) e^{j\left(2\pi f_n t + \phi_n(t)\right)} \quad (7)$$

where $f_n = f_0 + n\Delta f$

and $A_n(t), \phi_n(t), f_n$ are amplitude, phase, carrier frequency of n-th carrier, respectively.

Multi-carrier Equivalent Implementation by using IDFT

- Then we sample the signal at a sampling frequency $1/\Delta t$, and $A_n(t)$ and $\varphi_n(t)$ becomes:

$$\phi_n(t) = \phi_n \tag{8}$$

$$A_n(t) = A_n \tag{9}$$

$$S_s(k\Delta t) = \frac{1}{N}\sum_{n=0}^{N-1} A_n e^{j\left(2\pi(f_0 + n\Delta f)k\Delta t + \phi_n\right)} \tag{10}$$

- Then the sampled signal can be expressed as:

$$S_s(k\Delta t) = \frac{1}{N}\sum_{n=0}^{N-1}\left(A_n e^{j\left(2\pi f_0 k\Delta t + \phi_n\right)}\right)\cdot e^{j2\pi nk\Delta f\Delta t} \tag{11}$$

Multi-carrier Equivalent Implementation by using IDFT

- The inverse discrete Fourier transform (IDFT) is defined as the following:

$$f(k\Delta t) = \frac{1}{N}\sum_{n=0}^{N-1} F(n\Delta f) e^{j2\pi nk/N} \qquad (12)$$

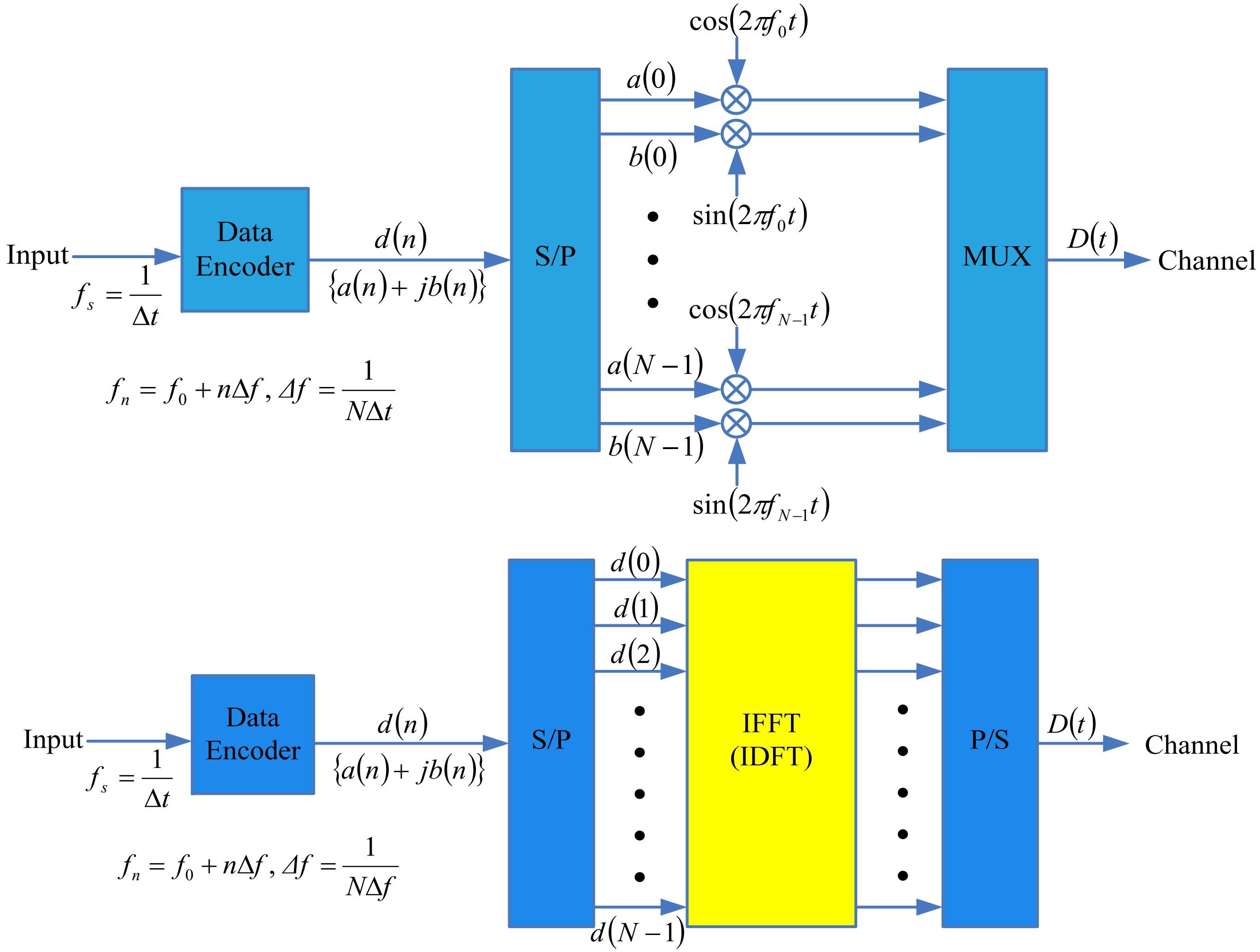
- Comparing eq.(11) and eq.(12), the condition must be satisfied in order to make eq.(11) an inverse Fourier transform relationship:

$$\Delta f = \frac{1}{N\Delta t} \qquad (13)$$

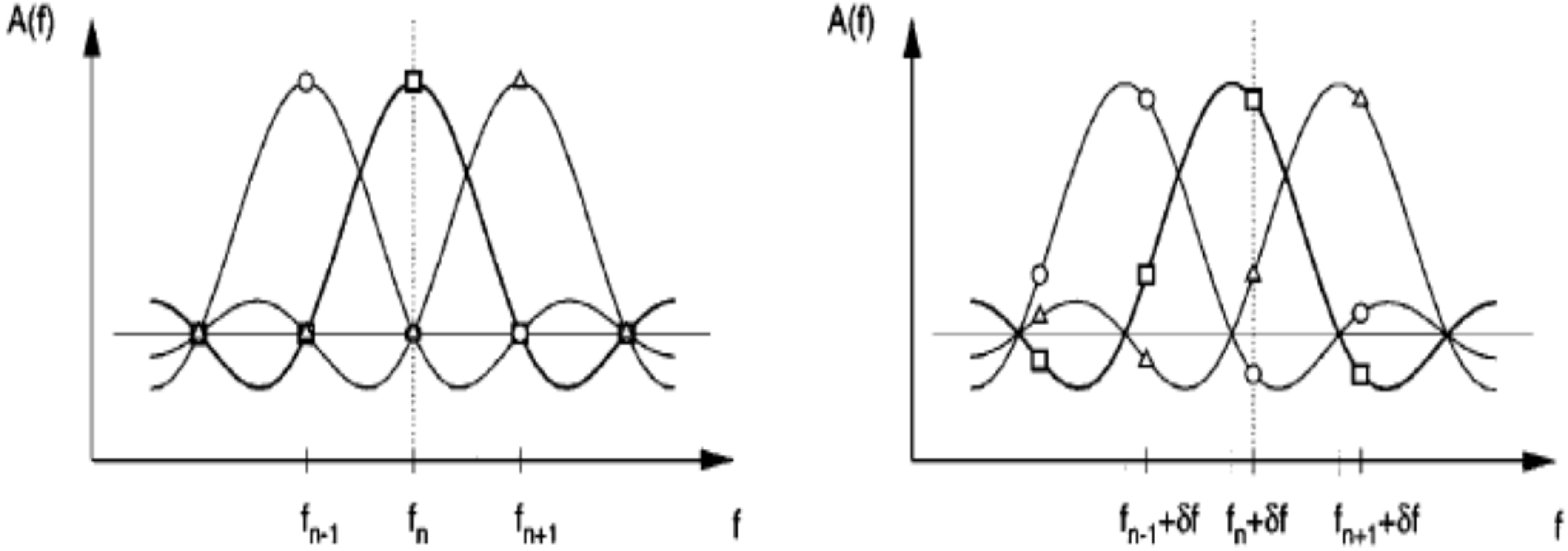
Multi-carrier Equivalent Implementation by using IDFT

- If eq.(13) is satisfied,
  - $A_n e^{j(2\pi f_0 k\Delta t+\phi_n)}$ is the frequency domain signal
  - $S_s(k\Delta t)$ is the time domain signal
  - Δf is the sub-channel spacing
  - $N\Delta t$ is the symbol duration in each sub-channel
- This outcome is the same as the result obtained in the system of Figure 1. Therefore IDFT can be used to generate an OFDM transmission signal.

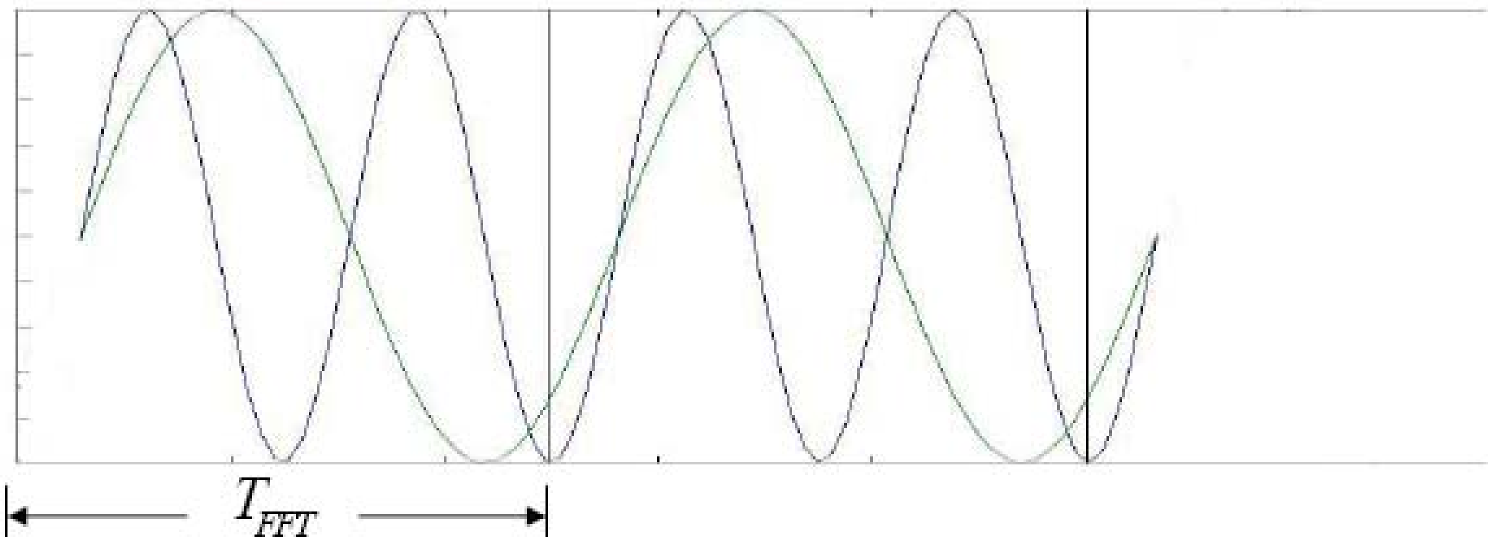
Multi-carrier Equivalent Implementation by using IDFT



Frequency Error Results in ICI



Synchronization Error Results in ICI

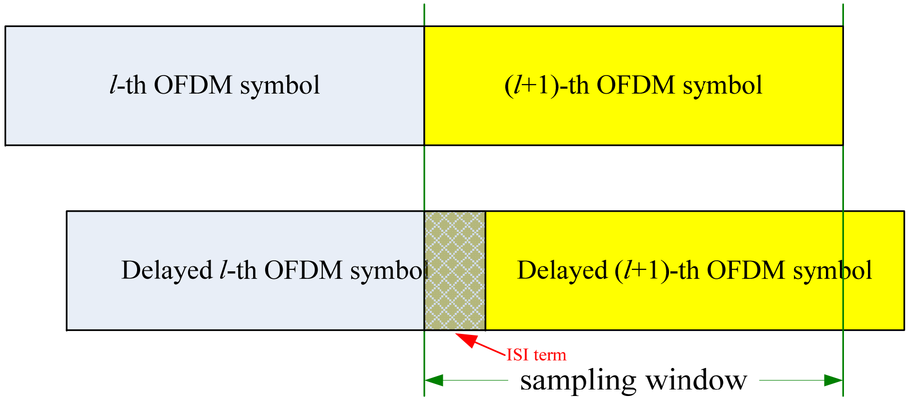


→ Not Orthogonal Any More.

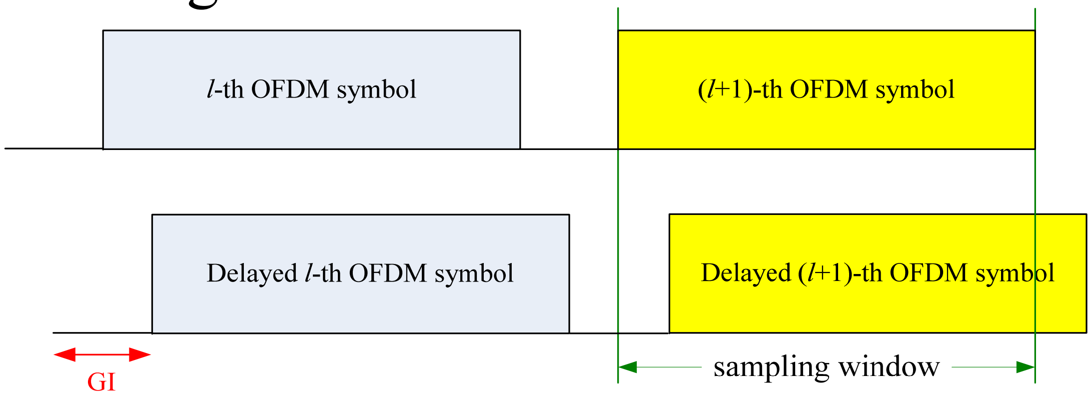
Cyclic Prefix (CP)

Cyclic Prefix

- In multipath channel, delayed replicas of previous OFDM signal lead to ISI between successive OFDM signals.

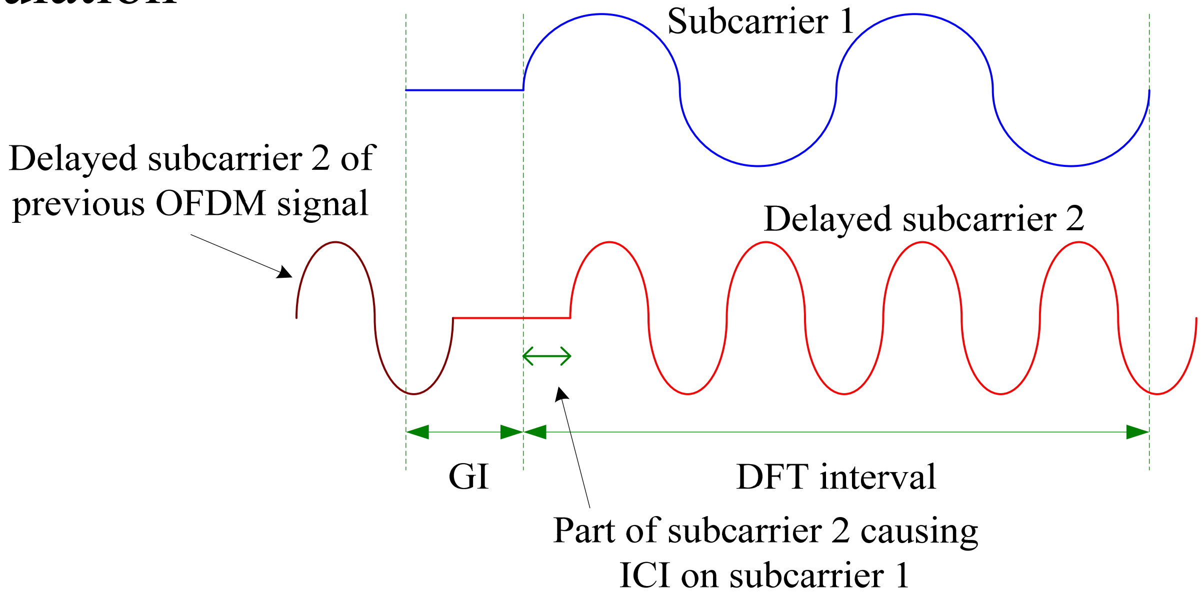


- Solution : Insert a guard interval between successive OFDM signals.



Cyclic Prefix

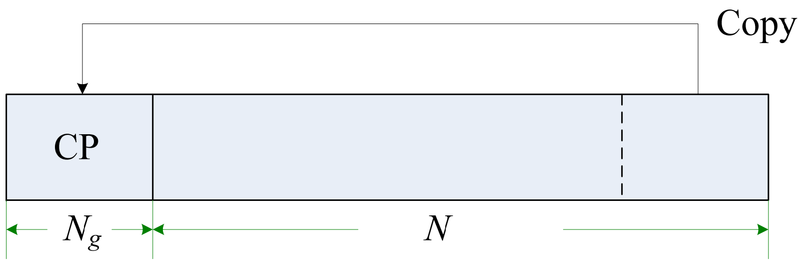
- Guard interval leads to intercarrier interference (ICI) in OFDM demodulation

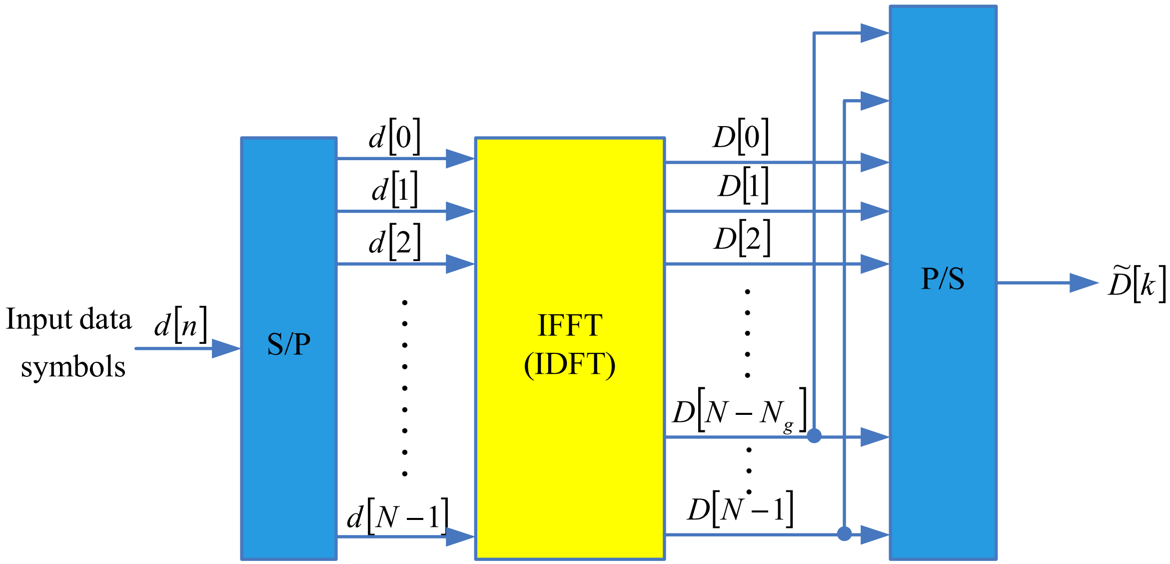


- In DFT interval, difference between two subcarriers does not maintain integer number of cycles ➔ loss of orthogonality.
- Delayed version of subcarrier 2 causes ICI in the process of demodulating subcarrier 1.

Cyclic Prefix

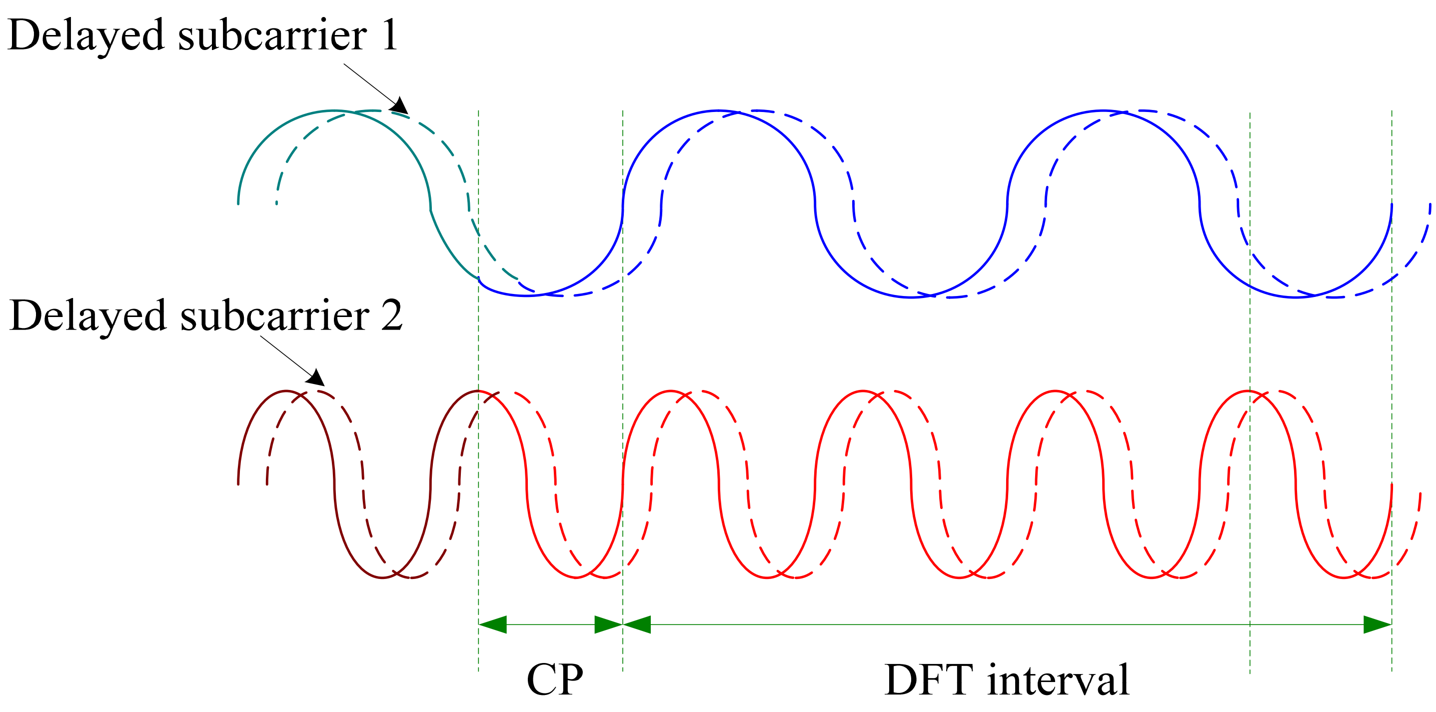
- Cyclic prefix (CP) : A copy of the last part of OFDM signal is attached to the front of itself.





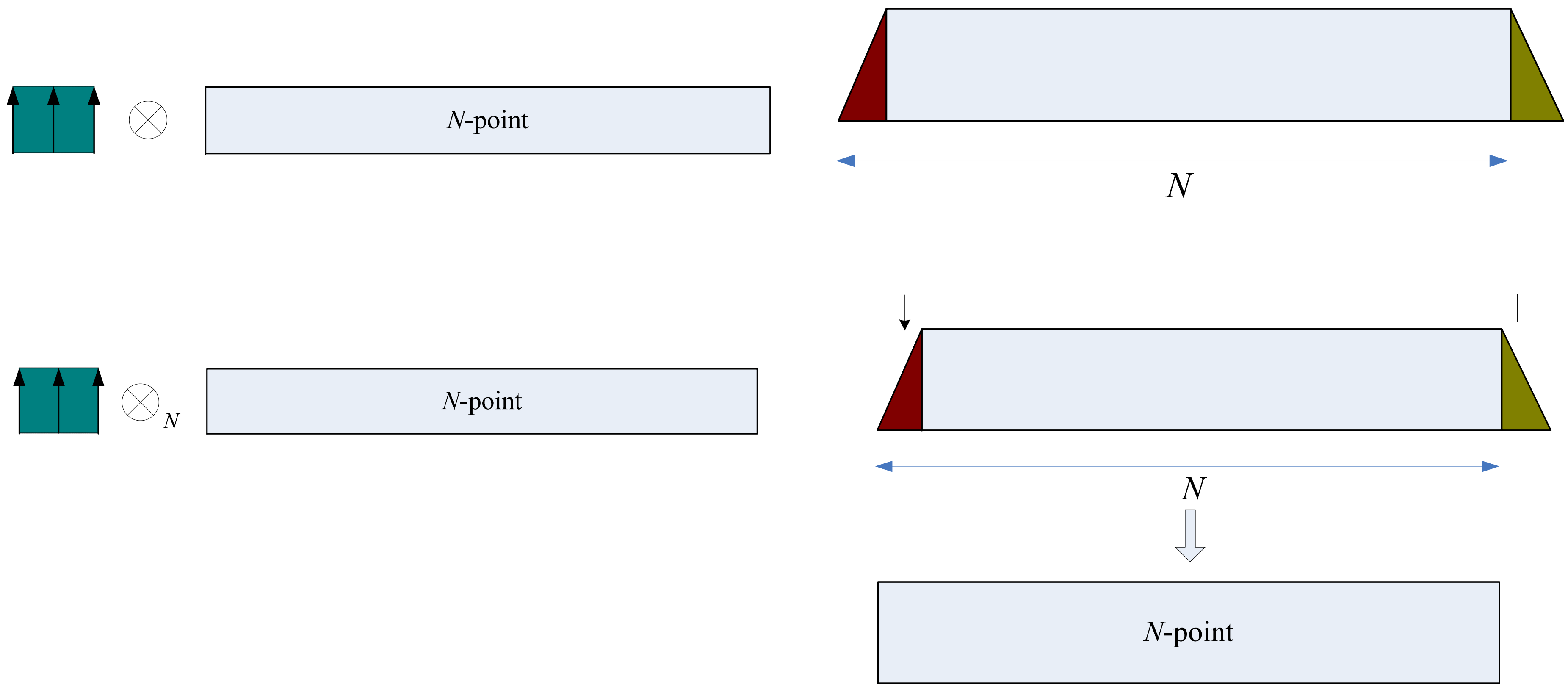
Cyclic Prefix

- All delayed replicas of subcarriers always have an integer number of cycles within DFT interval → no ICI



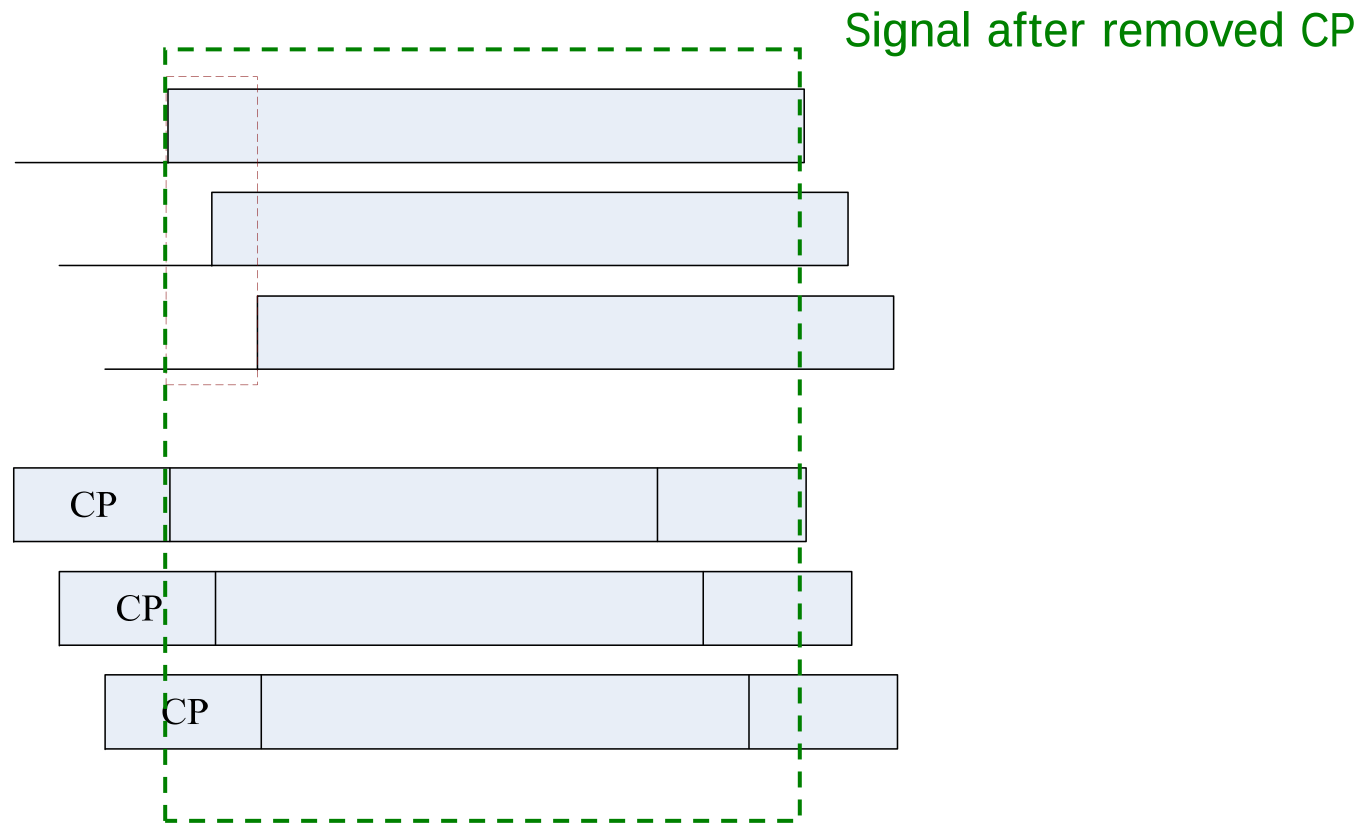
Cyclic Prefix

- Linear convolution vs. circular convolution



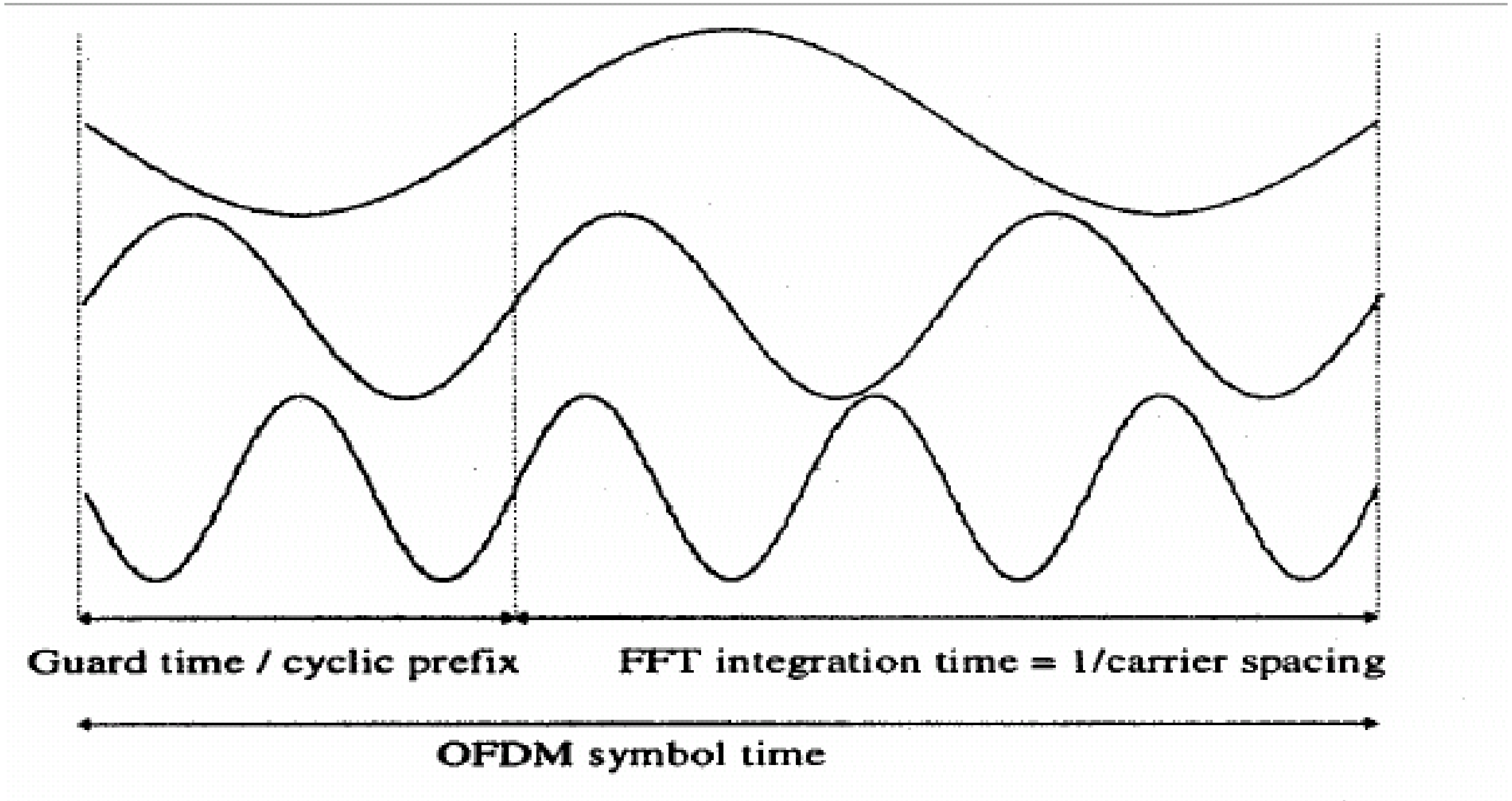
Cyclic Prefix

- Channel effect with cyclic prefix

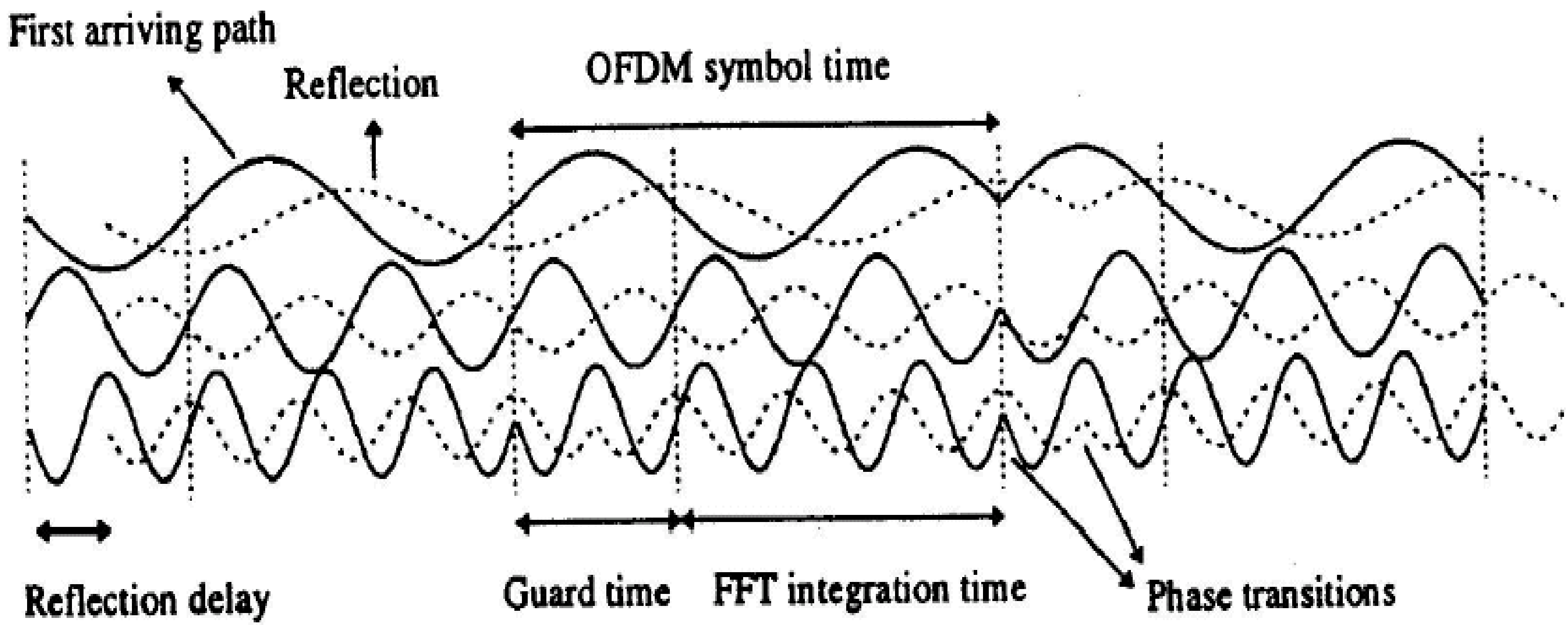


Cyclic Prefix

- Time-Domain Explanation



Cyclic Prefix



Cyclic Prefix

- Spectrum of channel response $h[n]$ with length L_h (smaller than N_g)

$$H_k = FFT\{h[n]\}$$

- Received complete OFDM signal

$$\tilde{r}[n] = \tilde{D}[n] \otimes h[n],\ 0 \le n \le N + N_g + L_h - 2$$

- Received useful part $r[n]$

$$r[n] = D[n] \otimes_N h[n]$$

where $\otimes_N$ is N-point circular convolution (due to CP)

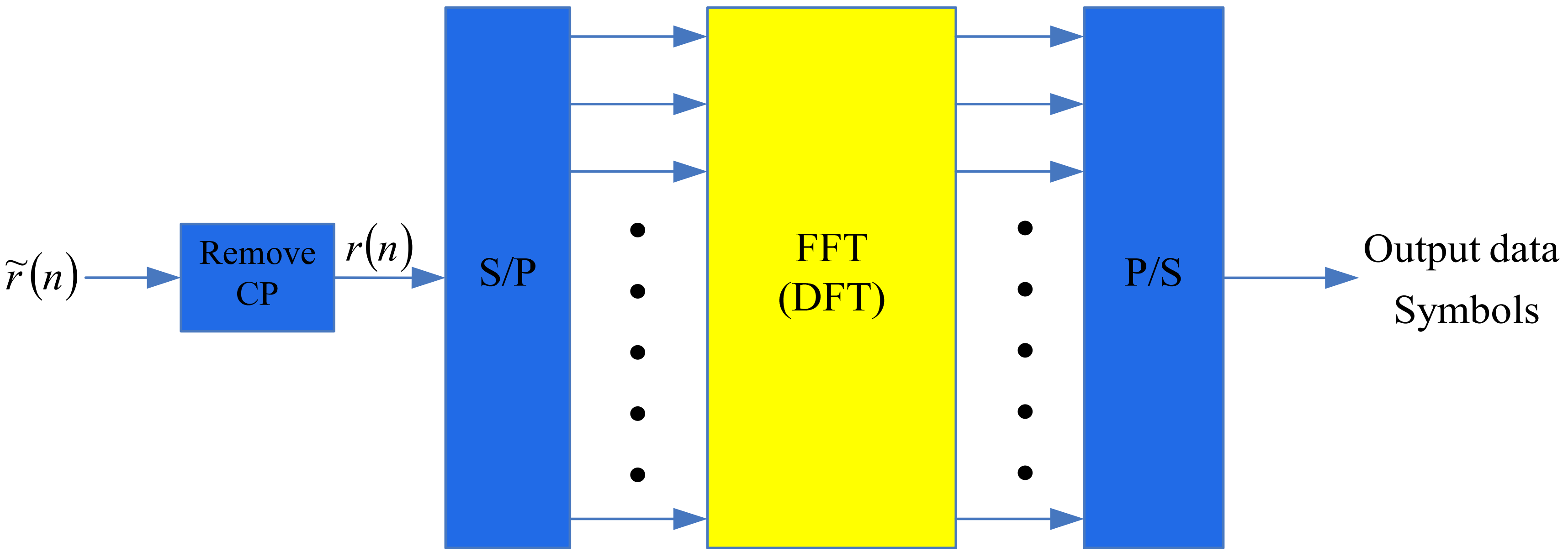
- Received symbol at k-th subcarrier

$$Y_k = FFT\{r[n]\} = FFT\{D[n] \otimes_N h[n]\} = X_k H_k$$

$$\Rightarrow X_k = \frac{Y_k}{H_k}$$

"Useful property for OFDM system to reduce complexity of channel equalization"

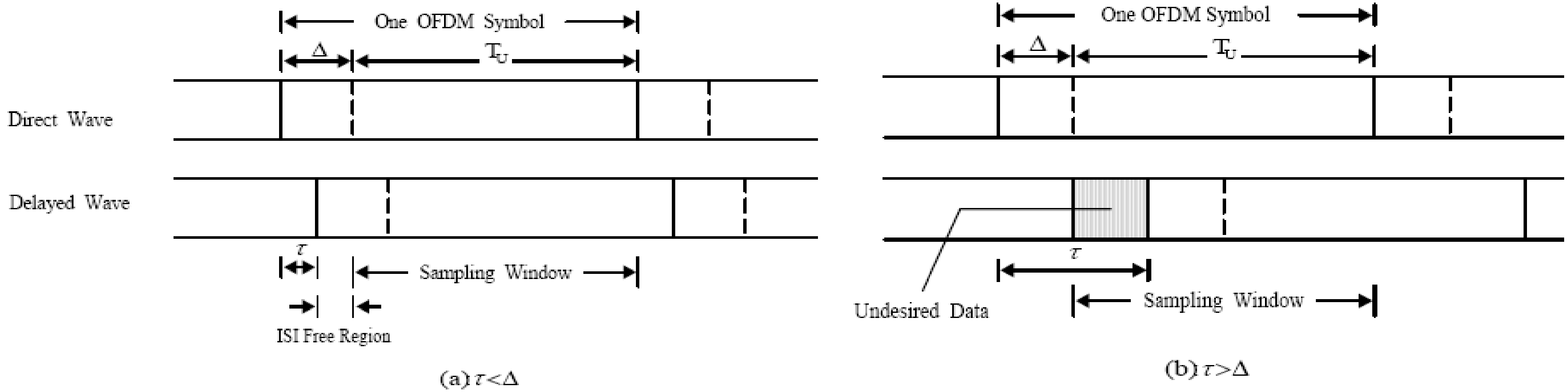
Cyclic Prefix (OFDM Receiver)



Cyclic Prefix

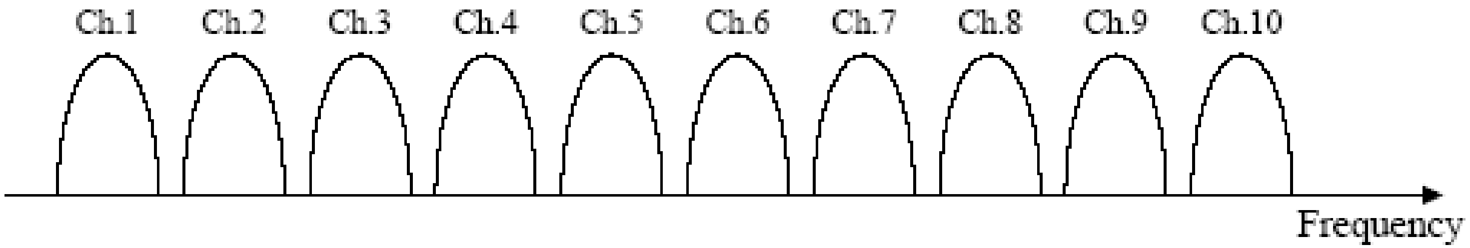
- One of the most important reasons to do OFDM is the efficient way it deals with multipath delay spread.
- To eliminate inter-symbol interference (ISI) almost completely, a guard time is introduced for each OFDM symbol.

(The guard time is chosen larger than the delay spread)

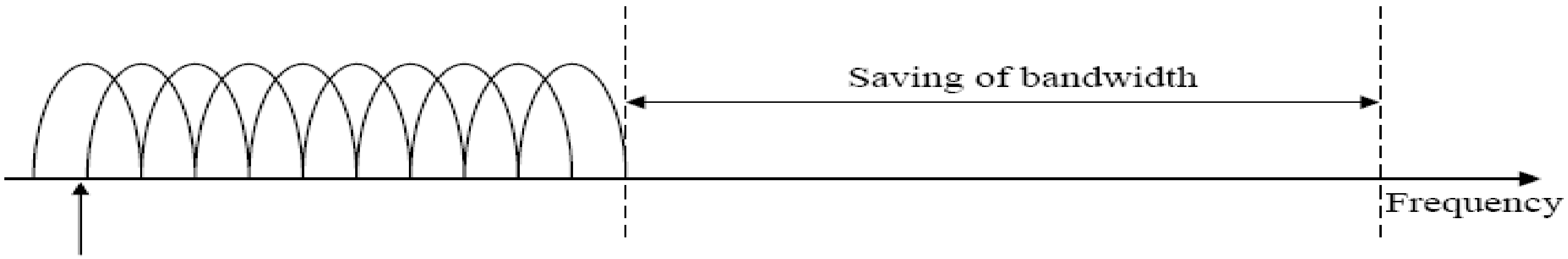


Bandwidth Efficiency

- In a classical parallel system, the channel is divided into *N* non-overlapping sub-channels to avoid inter-carrier interference (ICI).



- The diagram for bandwidth efficiency of OFDM system is shown below:



Summary

- The advantage of the FFT-based OFDM system :
  - The use of IFFT/FFT can reduce the computation complexity.
  - The orthogonality between the adjacent sub-carriers will make the use of transmission bandwidth more efficient.
  - The guard interval is used to resist the inter-symbol interference (ISI).
  - The main advantage of the OFDM transmission technique is its high performance even in frequency selective channels.
- The drawbacks of the OFDM system :
  - It is highly vulnerable to synchronization errors.
  - Peak to Average Power Ratio (PAPR) problems.