**Chapter5**

**Problem 1:(5.1)**

1. Show that the characteristic function of a Gaussian random variable *X* of mean  and variance  is



1. Using the result of part (a), show that the  central moment of this Gaussian random variable is



**Problem 2: (5.2)**

A Gaussian-distributed random variable X of zero mean and variance  is transformed by a piecewise-linear rectifier characterized by the input-output relation (see Figure P5.2):

$$Y=\left\{\begin{array}{c} X, \&X\geq 0\\0, \&X<0\end{array}\right.$$

The probability density function of the new random variable Y is described by



a. Explain the physical reasons for the functional form of this result.

b. Determine the value of the constant k by which the delta function is weighted.



**Problem 3:(5.5)**

For a complex random process , define the autocorrelation function as



where \* represents complex conjugation. Derive the properties of this complex autocorrelation corresponding to

(a) 

(b) 

(c) 

**Problem 4:(5.6)**

For the complex random process  where  and  are real-valued random processes given by



and



where  and  are uniformly distributed over . What is the autocorrelation of ? Suppose? Suppose?

**[HINT]**

1. The mean-square value of the process may be obtained from  simply by putting .



1. The autocorrelation function  is an even function of.



1. The autocorrelation function  has its maximum magnitude at .



**Problem 5:(5.8)**

Prove the following two properties of the autocorrelation function $R\_{X}(τ)$ of a random process $X(t)$:

1. If $X(t)$ contains a dc component equal to , then $R\_{X}(τ)$ will contain a constant component equal to $A^{2}$.
2. If $X(t)$ contains a sinusoidal component, then $R\_{X}(τ)$ will also contain a sinusoidal component of the same frequency.

**Problem 6:(5.12)**

Consider a pair of wide-sense stationary random processes  and . Show that the cross-correlations  and  of these processes have the following properties:

(a) 

(b) 

**Problem 7:(5.13)**

Consider two linear filters connected in cascade as in Figure P5.13. Let  be a wide-sense stationary process with autocorrelation function . The random process appearing at the first filter output is  and that at the second filter output is  $.$

1. Find the autocorrelation function of .
2. Find the cross-correlation function  of  and.

****

**Problem 8:(5.14)**

A wide-sense stationary random process  is applied to a linear time invariant filter of impulse response , producing an output .

1. Show that the cross-correlation function  of the output  and the input  is equal to the impulse response  convolved with the autocorrelation function  of the input, as shown by



Show that the second cross-correlation function  equals



1. Find the cross-spectral densities  and.
2. Assuming that  is a white noise process with zero mean and power spectral density, show that



Comment on the practical significance of this result.

**Problem 9:(5.23)**

A stationary, Gaussian process  with zero mean and power spectral density  is applied to a linear filter whose impulse response  is shown in Figure P5.23. A sample *Y* is taken of the random process at the filter output at time *T*.

1. Determine the mean and variance of *Y*.
2. What is the probability density function of *Y* ?

