**Exercise chapter 4**

**[1]**



**[2]**



**[3]**





**[4]**





**[5]**

1. Since  the dimensionality of the signal space is two.

2. As a basis of the signal space we consider the functions:



 The vector representation of the signal is:



3. The signal constellation is depicted in the next figure:



4. The three possible output of the matched filters, corresponding to the three possible transmitted signals are  and , where  are zero mean Gaussian random variable with variance . If all the signals are equiprobable the optimum decision rule selects the signal that maximize the metric:



or since  is the same for all ,

Thus the optimal decision region  for  is the set of points , such that  and . Since 

, the previous conditions are written as



Similarly we find that  is the set of points  that satisfy  and  is the region such that . The regions  are shown in the next figure.



5. If the signals are equiprobable then:



When  is transmitted then  and therefore,  is written as: 

Since,  are zero mean statistically independent Gaussian random variables, each with variance , the random variables  and  are zero-mean Gaussian with variance . Hence:



When  is transmitted then  and therefore:



Similarly from the symmetry of the problem, we obtain:



Since  is momononically decreasing, we obtain:



and therefore, the probability of error  is larger than  and .Hence, the message  is more vulnerable to errors. the reason for that is that it has both threshold lines close to it, while the other two signals have one of the threshold lines further away.

**[6]**

1. The transmitted energy is:



2. The correlation coefficient for the two signals is:



 Hence, the bit error probability for coherent detection is:



3. The bit error probability for non-coherent detection is given by:



 where  is the generalized Marcum Q function and

