

### Solution 1.

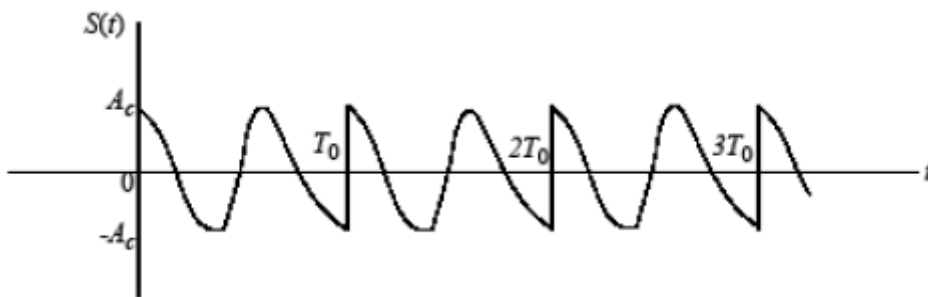
根據頻率來畫圖。瞬時頻率求法： $f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$

$$(1). s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

$$\text{instantaneous angle } \theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$\text{instantaneous frequency } f_i(t) = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}$$

由上式可以知道 PM signal 的頻率跟  $m(t)$  的斜率呈現正比關係。(訊號斜率為 1，故頻率不變。)

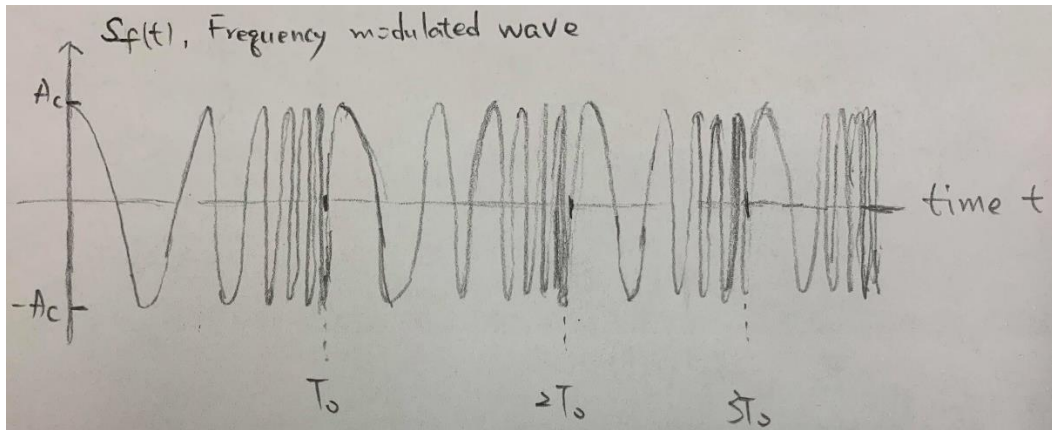


$$(2). s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]$$

$$\text{instantaneous angle } \theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$$

$$\text{instantaneous frequency } f_i(t) = f_c + k_f m(t)$$

由上式可以知道 FM signal 的頻率跟  $m(t)$  振幅呈現正比關係。(訊號愈來愈大，頻率也會愈來愈大)



Solution 2.

(a)

Problem 4.5

(a) The phase-modulated wave is

$$\begin{aligned}
 s(t) &= A_c \cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)] \\
 &= A_c \cos[2\pi f_c t + \beta_p \cos(2\pi f_m t)] \\
 &= A_c \cos(2\pi f_c t) \cos[\beta_p \cos(2\pi f_m t)] - A_c \sin(2\pi f_c t) \sin[\beta_p \cos(2\pi f_m t)]
 \end{aligned}$$

If  $\beta_p \leq 0.5$ , then

$$\cos[\beta_p \cos(2\pi f_m t)] \approx 1$$

$$\sin[\beta_p \cos(2\pi f_m t)] \approx \beta_p \cos(2\pi f_m t)$$

Hence, we may rewrite Eq. (1) as

$$\begin{aligned}
 s(t) &\approx A_c \cos(2\pi f_c t) - \beta_p A_c \sin(2\pi f_c t) \cos(2\pi f_m t) \\
 &= A_c \cos(2\pi f_c t) - \frac{1}{2} \beta_p A_c \sin[2\pi(f_c + f_m)t] \\
 &\quad - \frac{1}{2} \beta_p A_c \sin[2\pi(f_c - f_m)t]
 \end{aligned}$$

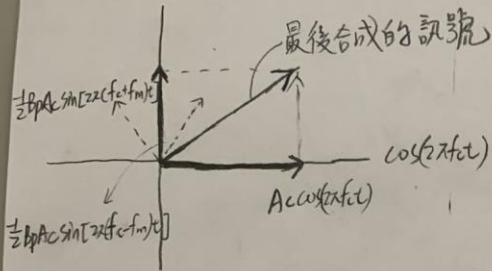
The spectrum of  $s(t)$  is therefore

$$\begin{aligned}
 S(f) &\approx \frac{1}{2} A_c [\delta(f - f_c) + \delta(f + f_c)] \\
 &\quad - \frac{1}{4j} \beta_p A_c [\delta(f - f_c - f_m) - \delta(f + f_c + f_m)] \\
 &\quad - \frac{1}{4j} \beta_p A_c [\delta(f - f_c + f_m) - \delta(f + f_c - f_m)]
 \end{aligned}$$

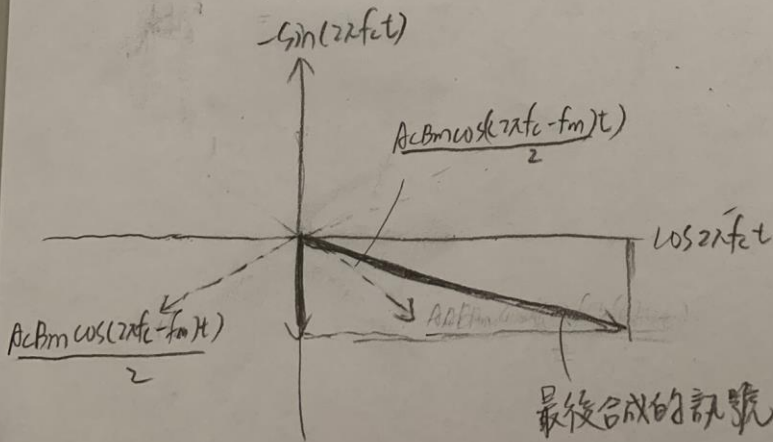
(b)

2(b) phasor diagram:

$$\begin{aligned} PM: s(t) &\approx A_c \cos(2\pi f_c t) - B_p A_c \sin(2\pi f_c t) \cos(2\pi f_m t) \\ &= A_c \cos(2\pi f_c t) - \frac{1}{2} B_p A_c \sin[2\pi(f_c + f_m)t] - \frac{1}{2} B_p A_c \sin[2\pi(f_c - f_m)t] \\ &\quad - \sin(2\pi f_c t) \end{aligned}$$



$$\begin{aligned} NBFM: s(t) &= A_c \cos(2\pi f_c t) - A_c \left\{ 2\pi k_f \int_0^t m(t) dt \right\} \sin 2\pi f_c t \\ &= A_c \cos(2\pi f_c t) - A_c \left\{ 2\pi k_f \int_0^t A_m \cos(2\pi f_m t) dt \right\} \sin 2\pi f_c t \\ &= A_c \cos(2\pi f_c t) - A_c A_m \frac{2\pi k_f}{2\pi f_m} \sin 2\pi f_c t \Big|_0^t \sin 2\pi f_c t \\ &= A_c \cos(2\pi f_c t) - A_c B_m \sin 2\pi f_m t \sin 2\pi f_c t \\ &= A_c \cos(2\pi f_c t) - A_c B_m \left( \frac{-\cos(2\pi(f_m + f_c)t) + \cos(2\pi(f_c - f_m)t)}{2} \right) \\ &\quad - \sin(2\pi f_c t) \end{aligned}$$



Solution 3.

$$s(t) = A_c \cos(\theta(t))$$

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

Let  $\beta < 0.3$  for  $m(t) = \cos(2\pi f_m t)$ .

$$\therefore s(t) = A_c \cos(2\pi f_c t + \beta m(t))$$

$$= A_c [\cos(2\pi f_c t) \cos(\beta \cos(2\pi f_m t)) - \sin(2\pi f_c t) \sin(\beta \cos(2\pi f_m t))]$$

for small  $\beta$ :

$$\cos(\beta \cos(2\pi f_m t)) \approx 1$$

$$\sin(\beta \cos(2\pi f_m t)) \approx \beta \cos(2\pi f_m t)$$

$$\therefore s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \cos(2\pi f_m t)$$

$$= A_c \cos(2\pi f_c t) - \beta \frac{A_c}{2} [\sin(2\pi(f_c + f_m)t) + \sin(2\pi(f_c - f_m)t)]$$