**Problem 1:(2.1)**

1. Find the Fourier transform of the half-cosine pulse shown in Figure P2.1a.
2. Apply the time-shifting property to the result obtained in part (a) to evaluate the spectrum of the half-sine pulse shown in Figure P2.1b.
3. What is the spectrum of a half-sine pulse having a duration equal to ?
4. What is the spectrum of the negative half-sine pulse shown in Figure P2.1c?
5. Find the spectrum of the single sine pulse shown in Figure P2.1d.



Fig. P2.1.

**Problem 2:(2.2)**

Evaluate the Fourier transform of the damped sinusoidal wave



where  is the unit step function.

**Problem 3:(2.3)**

Any function can be split unambiguously into an even part and an odd part, as shown by  The even part is defined by  and the odd part is defined by .

1. Evaluate the even and odd parts of a rectangular pulse defined by .
2. What are the Fourier transforms of these two parts of the pulse?

**Problem 4:(2.4)**

Determine the inverse Fourier transform of the frequency function  defined by the amplitude and phase spectra shown in P2.4.



Fig. P2.4.

**Problem 5:(2.6)**

The Fourier transform of a signal  is denoted by . Prove the following properties of the Fourier transform:

1. If a real signal  is an even function of time t, the Fourier transform  is purely real. If a real signal is an odd function of time t, the Fourier transform is purely imaginary.
2. , where  is the nth derivative of  with respect to .
3. .
4. **.**
5. **.**

**Problem 6:(2.8)**

Prove the following properties of the convolution process:

1. The commutative property:**.**
2. The associative property:****
3. The distributive property:****

**Problem 7:(2.9)**

Consider the convolution of two signals  and . Show that:

1. 
2. 

**Problem 8:(2.11)**

Evaluate the Fourier transform of the delta function by considering it as the limiting form of (1) a rectangular pulse of unit area, and (2) a sinc pulse of unit area.

**Problem 9:(2.12)**

The Fourier transform  of a signal  is defined by 

Determined the signal .

**Problem 10:(2.15)**

Let  and  be the input and output signals of a linear time-invariant filter. Using Rayleigh's energy theorem, show that if the filter is stable and the input signal  has finite energy, then the output signal  also has finite energy. That is, given that , then show that .

**Problem 11:(2.16)**

Evaluate the transfer function of a linear system represented by the block diagram shown in Figure P2.16.



Fig. P2.16.