Communication Systems

2016 Spring–Ch7 exercises solution

Problem 1:

(a) $g(t) = \text{sinc}(200t)$

The sinc pulse corresponds to a bandwidth $W = 100$ Hz. Hence, the Nyquist rate is 200 Hz, and the Nyquist rate is $1/200$ seconds.

(b) $g(t) = \text{sinc}^2(200t)$

This signal may be viewed as the product of the sinc pulse $\text{sinc}(200t)$ with itself. Since multiplication in the time domain corresponds to convolution in the frequency domain, we find that the signal $g(t)$ has a bandwidth equal to twice that of the sinc pulse $\text{sinc}(200t)$, that is , 200 Hz. The Nyquist rate of $g(t)$ is therefore 400 Hz, and the Nyquist interval is $1/400$ seconds.

(c) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

The bandwidth of $g(t)$ is determined by the highest frequency component of $\text{sinc}(200t)$ or $\text{sinc}^2(200t)$, whichever one is the largest. With the bandwidth (i.e., highest frequency component of) the sinc pulse $\text{sinc}(200t)$ equal to 100 Hz and that of the square sinc pulse $\text{sinc}^2(200t)$ equal to 200 Hz, it follows that the bandwidth of $g(t)$ is 200Hz. Correspondingly, the Nyquist rate of $g(t)$ is 400 Hz, and its Nyquist interval is $1/400$ seconds.

Problem 2:

(a) The sampling interval is $T_s = 125\ \mu$s. There are 24 channels and 1 sync pulse, so the time allotted to each channel is $T_c = T_s/25 = 5\ \mu$s. The pulse duration is $1\ \mu$s, so the time between pulses is $4\ \mu$s.

(b) If sampled at the Nyquist rate, 6.8 kHz, then $T_s = 147\ \mu$s, $T_c = 5.88\ \mu$s, and the time between pulses is $4.88\ \mu$s.

Problem 3:

(a) Let the message bandwidth be *W*. Then, sampling the message signal at its Nyquist rate, and using an *R*-bit code to represent each sample of the message signal, we find that the bit duration is

$$T_b = \frac{T_s}{R} = \frac{1}{2WR}$$

The bit rate is

$$\frac{1}{T_b} = 2WR$$

The maximum value of message bandwidth is therefore

$$W_{\max} = \frac{50 \times 10^6}{2 \times 7} = 3.57 \times 10^6 \text{ Hz}$$

(b) The output signal-to-quantizing noise ratio is given by

$$10 \log_{10} (\text{SNR})_0 = 1.8 + 6R$$
$$= 1.8 + 6 \times 7$$
$$= 43.8 \text{ dB}$$

Problem 4:

The modulating wave is

$$m(t) = A_m \cos(2\pi f_m t)$$

The slope of $m(t)$ is

$$\frac{dm(t)}{dt} = -2\pi f_m A_m \sin(2\pi f_m t)$$

The maximum slope of $m(t)$ is equal to $2\pi f_m A_m$.

The maximum average slope of the approximating signal $m_a(t)$ producted by the delta modulation is δ / T_s, where δ is the step size and T_s is the sampling period. The limiting value of A_m is therefore given by

$$2\pi f_m A_m > \frac{\delta}{T_s}$$

or

$$A_m > \frac{\delta}{2\pi f_m T_s}$$

Assuming a load of 1 ohm, the transmitted power is $A_m^2 / 2$. Therefore, the maximum power that may be transmitted without slope-overload distortion is equal to $\delta^2 / 8\pi^2 f_m^2 T_s^2$.

Problem 5:

$$f_s = 10 f_{\text{Nyquist}}$$
$$f_{\text{Nyquist}} = 6.8 \text{ kHz}$$
$$f_s = 10 \times 6.8 \times 10^3 = 6.8 \times 10^4 \text{ Hz}$$
$$\frac{\Delta}{T_s} \geq \max \left| \frac{dm(t)}{dt} \right|$$

For the sinusoidal signal $m(t) = A_m \sin(2\pi f_m t)$, we have

$$\frac{dm(t)}{dt} = 2\pi f_m A_m \cos(2\pi f_m t)$$

Hence,

$$\left|\frac{dm(t)}{dt}\right|_{\max} = \left|2\pi f_m A_m\right|_{\max}$$

or, equivalently,

$$\frac{\Delta}{T_s} \geq \left|2\pi f_m A_m\right|_{\max}$$

Therefore,

$$\left|A_m\right|_{\max} = \frac{\Delta}{T_s \times 2\pi \times f_m} = \frac{\Delta f_s}{2\pi \times f_m} = \frac{0.1 \times 6.8 \times 10^4}{2\pi \times 10^3} = 1.08\text{V}$$