Communication Systems
2019–Ch5 exercises

Problem 1:

Show that the characteristic function of a Gaussian random variable X of mean μ_X and variance σ_X^2 is

$$\phi_X(\upsilon) = \exp(j\upsilon\mu_X - \tfrac{1}{2}\upsilon^2\sigma_X^2)$$

Problem 2:

For a complex random process $Z(t)$, define the autocorrelation function as

$$R_Z(\tau) = \mathrm{E}[Z^*(t)Z(t+\tau)]$$

where * represents complex conjugation. Derive the properties of this complex autocorrelation corresponding to

(a) $R_Z(0) = \mathrm{E}[|Z(t)|^2]$

(b) $R_Z(-\tau) = R_Z^*(\tau)$

(c) $\left|\mathrm{Re}\{R_Z(\tau)\}\right| \le R_Z(0)$

Problem 3:

Consider a pair of wide-sense stationary random processes $X(t)$ and $Y(t)$. Show that the cross-correlations $R_{XY}(\tau)$ and $R_{YX}(\tau)$ of these processes have the following properties:

(a) $R_{XY}(\tau) = R_{YX}(-\tau)$

(b) $|R_{XY}(\tau)| \le \dfrac{1}{2}[R_X(0) + R_Y(0)]$

Problem 4:

A stationary, Gaussian process $X(t)$ with zero mean and power spectral density $S_X(f)$ is applied to a linear filter whose impulse response $h(t)$ is shown in Figure P5.23. A sample *Y* is taken of the random process at the filter output at time *T*.

(a) Determine the mean and variance of *Y*.

(b) What is the probability density function of *Y* ?

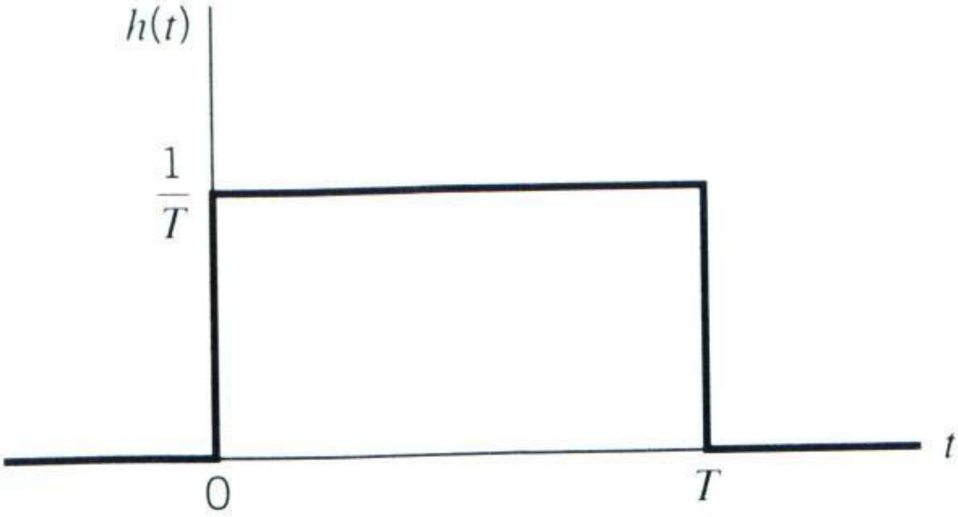

Figure P5.23