2018-Chapter 2 solution

Problem 1:

$G(f)=\frac{1}{2}+\frac{1}{2}\text{sgn}(f)$, by duality: $G(f)\Leftrightarrow\frac{1}{2}\delta(-t)-\frac{1}{j2\pi t}$

$$\therefore g(t)=\frac{1}{2}\delta(t)+\frac{j}{2\pi t}$$

Problem 2:

$$\begin{aligned} g(t) &= \exp(-t)\sin(2\pi f_c t)\text{u}(t) \\ &= \left[\exp(-t)\text{u}(t)\right]\sin(2\pi f_c t) \end{aligned}$$

$$\begin{aligned} \therefore G(f) &= \frac{1}{1+j2\pi f} * \left[\frac{1}{2j}\left(\delta(f-f_c)-\delta(f+f_c)\right)\right] \\ &= \frac{1}{2j}\left[\frac{1}{1+j2\pi(f-f_c)}-\frac{1}{1+j2\pi(f+f_c)}\right] \end{aligned}$$

Problem 3:

$$H(f)=X(-f)\exp(-j2\pi fT)$$

$$\begin{aligned} X(f) &= \frac{A}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]*T\text{sinc}(fT)\exp(-j2\pi f\frac{T}{2}) \\ &= \frac{AT}{2}\left[\text{sinc}\left(T(f-f_c)\right)+\text{sinc}\left(T(f+f_c)\right)\right]\exp(-j\pi fT) \end{aligned}$$

Let $f_c=\frac{N}{T}$ for N large

$$\begin{aligned} Y(f) &= H(f)X(f) \\ &= X(-f)\exp(-j2\pi fT)\exp(-j\pi fT)\frac{AT}{2}\left[\text{sinc}\left(T(f-f_c)\right)+\text{sinc}\left(T(f+f_c)\right)\right] \\ &= \exp(-j2\pi fT)\frac{A^2T^2}{4}\left[\text{sinc}\left(T(f-f_c)\right)+\text{sinc}\left(T(f+f_c)\right)\right]\left[\text{sinc}\left(T(-f-f_c)\right)+\text{sinc}\left(T(-f+f_c)\right)\right] \\ &= \exp(-j2\pi fT)\frac{A^2T^2}{4}\left[\text{sinc}(-fT-N)+\text{sinc}(-fT+N)\right]\left[\text{sinc}(fT-N)+\text{sinc}(fT+N)\right] \end{aligned}$$