3.1 A carrier wave of frequency 1 MHz is modulated 50 percent tuned to the carrier frequency and has a Q-factor of 175. Deter-
cent by a sinusoidal frequency 5 kHz. The resulting AM signal is examine the modulated signal after transmission through this circuit.

3.4 Figure P3.4 shows the circuit diagram of a square-law modulator. The signal applied to the nonlinear device is relatively weak, such that it can be represented by a square law:

\[ v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \]

where \( a_1 \) and \( a_2 \) are constants, \( v_1(t) \) is the input voltage, and \( v_2(t) \) is the output voltage. The input voltage is defined by

\[ v_1(t) = A_c \cos(2\pi f_c t) + m(t) \]

where \( m(t) \) is a message signal and \( A_c \cos(2\pi f_c t) \) is the carrier wave.

(a) Evaluate the output voltage \( v_2(t) \).

(b) Specify the frequency response that the tuned circuit in Figure P3.4 must satisfy in order to generate an AM signal with \( f_c \) as the carrier frequency.

(c) What is the amplitude sensitivity of this AM signal?

3.6 Consider a square-law detector, using a nonlinear device whose transfer characteristic is defined by

\[ v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \]

where \( a_1 \) and \( a_2 \) are constants, \( v_1(t) \) is the input, and \( v_2(t) \) is the output. The input consists of the AM wave

\[ v_1(t) = A_c [1 + k_m m(t)] \cos(2\pi f_c t) \]

(a) Evaluate the output \( v_2(t) \).

(b) Find the conditions for which the message signal \( m(t) \) may be recovered from \( v_2(t) \).

3.7 The AM signal

\[ s(t) = A_c [1 + k_m m(t)] \cos(2\pi f_c t) \]

is applied to the system shown in Figure P3.7. Assuming that \( k_m m(t) < 1 \) for all \( t \) and the message signal \( m(t) \) is limited to the interval \(-W \leq f \leq W\), and that the carrier frequency \( f_c > 2W \), show that \( m(t) \) can be obtained from the square-root output \( v_2(t) \).

Figure P3.7

3.9 Figure P3.9 shows the circuit diagram of a balanced modulator. The input applied to the top AM modulator is \( m(t) \), whereas that applied to the lower AM modulator is \(-m(t)\); these two modulators have the same amplitude sensitivity. Show that the output \( s(t) \) of the balanced modulator consists of a DSB-SC modulated signal.

3.11 A DSB-SC modulated signal is demodulated by applying it to a coherent detector.

(a) Evaluate the effect of a frequency error \( \Delta f \) in the local carrier frequency of the detector, measured with respect to the carrier frequency of the incoming DSB-SC signal.

(b) For the case of a sinusoidal modulating wave, show that because of this frequency error, the demodulated signal exhibits beats at the error frequency. Illustrate your answer with a sketch of this demodulated signal.

3.12 Consider the DSB-SC signal

\[ s(t) = A_c \cos(2\pi f_c t) m(t) \]

where \( A_c \cos(2\pi f_c t) \) is the carrier wave and \( m(t) \) is the message signal. This modulated signal is applied to a square-law device characterized by

\[ y(t) = s^2(t) \]

The output \( y(t) \) is next applied to a narrow-band filter with a pass-band amplitude response of one, mid-band frequency \( 2f_c \), and bandwidth \( \Delta f \). Assume that \( \Delta f \) is small enough to treat the spectrum of \( y(t) \) as essentially constant inside the passband of the filter.

(a) Determine the spectrum of the square-law device output \( y(t) \).

(b) Show that the filter output \( y(t) \) is approximately sinusoidal, given by

\[ y(t) = \frac{A_c^2}{2} E \Delta f \cos(4\pi f_c t) \]

where \( E \) is the energy of the message signal \( m(t) \).
3.16 The single tone modulating signal \( m(t) = A_m \cos(2\pi f_m t) \) is used to generate the VSB signal
\[
s(t) = \frac{1}{2} A_m A_c \cos(2\pi f_c t) + \frac{1}{2} A_m A_c (1 - \alpha) \cos(2\pi (f_c - f_a) t)
\]
where \( \alpha \) is a constant, less than unity, representing the attenuation of the upper side frequency.

(a) If we represent this VSB signal as a quadrature carrier multiplex
\[
s(t) = A_{m1}(t) \cos(2\pi f_c t) + A_{m2}(t) \sin(2\pi f_c t)
\]

What is \( m_2(t) \)?

(b) The VSB signal, plus the carrier \( A_c \cos(2\pi f_c t) \), is passed through an envelope detector. Determine the distortion produced by the quadrature component, \( m_2(t) \).

(c) What is the value of constant \( \alpha \) for which this distortion reaches its worst possible condition?

3.21 The spectrum of a voice signal \( m(t) \) is zero outside the interval \( f_a \leq |f| \leq f_m \). In order to ensure communication privacy, this signal is applied to a scrambler that consists of the following cascade of components: a product modulator, a high-pass filter, a second product modulator, and a low-pass filter. The carrier wave applied to the first product modulator has a frequency equal to \( f_c \), whereas that applied to the second product modulator has a frequency equal to \( f_a + f_c \); both of them have unit amplitude. The high-pass and low-pass filters have the same cutoff frequency at \( f_c \).
Assume that \( f_c > f_a \).

(a) Derive an expression for the scrambler output \( s(t) \), and sketch its spectrum.

(b) Show that the original voice signal \( m(t) \) may be recovered from \( s(t) \) by using an unscrambler that is identical to the unit described above.

3.19 Figure P3.19 shows the block diagram of Weaver's method for generating SSB modulated waves. The message (modulating) signal \( m(t) \) is limited to the frequency band \( f_a \leq |f| \leq f_m \). The auxiliary carrier applied to the first pair of product modulators has a frequency \( f_0 \), which lies at the center of this band, as shown by
\[
f_0 = \frac{f_a + f_m}{2}
\]
The low-pass filters in the upper and lower branches are identical, each with a cutoff frequency equal to \( (f_a + f_m)/2 \). The carrier applied to the second pair of product modulators has a frequency \( f_c \) that is greater than \( (f_m - f_a)/2 \). Sketch the spectra at the various points in the modulator of Figure P3.19, and hence show that:

(a) For the lower sideband, the contributions of the upper and lower branches are of opposite polarity, and by adding them at the modulator output, the lower sideband is suppressed.

(b) For the upper sideband, the contributions of the upper and lower branches are of the same polarity, and by adding them, the upper sideband is transmitted.

(c) How would you modify the modulator of Figure P3.19, so that only the lower sideband is transmitted?