\[ s(t) = A_c [1 + k_a m(t)] \cos(2 \pi f_c t) \]

where \( m(t) = \sin(2 \pi f_s t) \) and \( f_s = 5 \text{ kHz} \) and \( f_c = 1 \text{ MHz} \).

\[ s(t) = A_c \left[ \cos(2 \pi f_c t) + \frac{k_a}{2} (\sin(2 \pi (f_c + f_s) t) + \sin(2 \pi (f_c - f_s) t)) \right] \]

\( s(t) \) is the signal before transmission.

The filter bandwidth is: \( BW = \frac{f_c}{Q} = \frac{10^6}{175} = 5714 \text{ Hz} \)

\( m(t) \) lies close to the 3dB bandwidth of the filter, \( m(t) \) is therefore attenuated by a factor of a half.

\[ m'(t) = 0.5 m(t) \quad \text{or} \quad k_a' = 0.5 k_a \]

\[ k_a' = 0.25 \]

The modulation depth is 0.25
Problem 3.4
Consider the square-law characteristic:

\[ v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \]  \hspace{1cm} (1)

where \( a_1 \) and \( a_2 \) are constants. Let

\[ v_1(t) = A_c \cos(2\pi f_c t) + m(t) \]  \hspace{1cm} (2)

Therefore substituting Eq. (2) into (1), and expanding terms:

\[ v_2(t) = a_1 A_c \left[ 1 + \frac{2a_2}{A_1} m(t) \right] \cos(2\pi f_c t) \]

\[ + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2(2\pi f_c t) \]  \hspace{1cm} (3)

The first term in Eq. (3) is the desired AM signal with \( k_a = 2a_2/a_1 \). The remaining three terms are unwanted terms that are removed by filtering.

Let the modulating wave \( m(t) \) be limited to the band \(-W \leq f \leq W\), as in Fig. 1(a). Then, from Eq. (3) we find that the amplitude spectrum \( |V_2(f)| \) is as shown in Fig. 1(b). It follows therefore that the unwanted terms may be removed from \( v_2(t) \) by designing the tuned filter at the modulator output of Fig. P2.2 to have a mid-band frequency \( f_c \) and bandwidth \( 2W \), which satisfy the requirement that \( f_c > 3W \).

![Figure 1](image-url)
Problem 3.6

Let

\[ v_1(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t) \]

(a) Then the output of the square-law device is

\[ v_2(t) = a_1 v_1 + a_2 v_1^2(t) \]

\[ = a_1 A_c [1 + k_a m(t)] \cos(2\pi f_c t) \]

\[ + \frac{1}{2} a_2 A_c^2 [1 + k_a m(t) + k_a^2 m^2(t)][1 + \cos(4\pi f_c t)] \]

(b) The desired signal, namely \( a_2 A_c^2 k_a m(t) \), is due to the \( a_2 v_1^2(t) \) - hence, the name "square-law detection". This component can be extracted by means of a low-pass filter. This is not the only contribution within the baseband spectrum, because the term \( 1/2 a_2 A_c^2 k_a^2 m^2(t) \) will give rise to a plurality of similar frequency components. The ratio of wanted signal to distortion is \( 2/k_a m(t) \). To make this ratio large, the percentage modulation, that is, \(|k_a m(t)|\) should be kept small compared with unity.
Problem 3.7

The squarer output is

\[ v_1(t) = A_c^2 [1 + k_d m(t)]^2 \cos^2 (2\pi f_c t) \]
\[ = \frac{A_c^2}{2} [1 + 2k_d m^2(t)][1 + \cos(4\pi f_c t)] \]

The amplitude spectrum of \( v_1(t) \) is therefore as follows, assuming that \( m(t) \) is limited to the interval \(-W \leq f \leq W\):

Since \( f_c > 2W \), we find that \( 2f_c - 2W > 2W \). Therefore, by choosing the cutoff frequency of the low-pass filter greater than \( 2W \), but less than \( 2f_c - 2W \), we obtain the output

\[ v_2(t) = \frac{A_c^2}{2} [1 + k_d m(t)]^2 \]

Hence, the square-rooter output is

\[ v_3(t) = \frac{A_c}{\sqrt{2}} [1 + k_d m(t)] \]

which, except for the dc component \( \frac{A_c}{\sqrt{2}} \), is proportional to the message signal \( m(t) \).

Problem 3.9

The two AM modulator outputs are

\[ s_1(t) = A_c [1 + k_d m(t)] \cos(2\pi f_c t) \]
\[ s_2(t) = A_c [1 + k_d m(t)] \cos(2\pi f_c t) \]

Subtracting \( s_2(t) \) from \( s_1(t) \):

\[ s(t) = s_2(t) - s_1(t) \]
\[ = 2k_d m(t) \cos(2\pi f_c t) \]

which represents a DSB-SC modulated wave.
Problem 3.11

(a) Multiplying the signal by the local oscillator gives:

\[ s_1(t) = A_c m(t) \cos(2\pi f_c t) \cos[2\pi (f_c + \Delta f)t] \]

\[ = \frac{A_c}{2} m(t) \{ \cos(2\pi \Delta ft) + \cos[2\pi (f_c + \Delta ft)] \} \]

Low pass filtering leaves:

\[ s_2(t) = \frac{A_c}{2} m(t) \cos(2\pi \Delta ft) \]

Thus the output signal is the message signal modulated by a sinusoid of frequency \( \Delta f \).

(b) If \( m(t) = \cos(2\pi f_m t) \),

then \( s_2(t) = \frac{A_c}{2} \cos(2\pi f_m t) \cos(2\pi \Delta ft) \)

\[ Y(f) = \frac{A_c^2}{2} \int_{-\infty}^{\infty} M(\lambda)M(f-\lambda)d\lambda + \frac{A_c^2}{4} \left[ \int_{-\infty}^{\infty} M(\lambda)M(f-2f_c-\lambda)d\lambda + \int_{-\infty}^{\infty} M(\lambda)M(f+2f_c-\lambda)d\lambda \right] \]

where \( M(f) = F[m(t)] \).
(b) At $f = 2f_c$, we have

\[ Y(2f_c) = \frac{A_c^2}{2} \int_{-\infty}^{\infty} M(\lambda)M(2f_c - \lambda)\, d\lambda \]

\[ + \frac{A_c^2}{4} \left[ \int_{-\infty}^{\infty} M(\lambda)M(-\lambda)\, d\lambda + \int_{-\infty}^{\infty} M(\lambda)M(4f_c - \lambda)\, d\lambda \right] \]

Since $M(-\lambda) = M^*(\lambda)$, we may write

\[ Y(2f_c) = \frac{A_c^2}{2} \int_{-\infty}^{\infty} M(\lambda)M(2f_c - \lambda)\, d\lambda \]

\[ + \frac{A_c^2}{4} \left[ \int_{-\infty}^{\infty} |M(\lambda)|^2\, d\lambda + \int_{-\infty}^{\infty} M(\lambda)M(4f_c - \lambda)\, d\lambda \right] \quad (1) \]

With $m(t)$ limited to $-W \leq f \leq W$ and $f_c > W$, we find that the first and third integrals reduce to zero, and so we may simplify Eq. (1) as follows

\[ Y(2f_c) = \frac{A_c^2}{4} \int_{-\infty}^{\infty} |M(\lambda)|^2\, d\lambda \]

\[ = \frac{A_c^2}{4} E \]

where $E$ is the signal energy (by Rayleigh’s energy theorem). Similarly, we find that

\[ Y(-2f_c) = \frac{A_c^2}{4} E \]

The band-pass filter output, in the frequency domain, is therefore defined by

\[ V(f) \approx \frac{A_c^2}{4} E \Delta f \{ \delta(f - 2f_c) + \delta(f + 2f_c) \} \]

Hence,

\[ v(t) \approx \frac{A_c^2}{4} E \Delta f \cos(4\pi f_c t) \]
3.16 (a)

\[ s(t) = \frac{1}{2} a \cdot A_m A_c \cos(2\pi f_m t + f_c t) + \frac{1}{2} (1-a) A_m A_c \cos(2\pi f_m t) \]

\[ s(t) = \frac{A_m A_c}{2} \left[ a(\cos(2\pi f_c t) \cos(2\pi f_m t) - \sin(2\pi f_c t) \sin(2\pi f_m t)) + (1-a)(\cos(2\pi f_c t) \cos(2\pi f_m t) + \sin(2\pi f_c t) \sin(2\pi f_m t)) \right] \]

\[ s(t) = \frac{A_m A_c}{2} \left[ \cos(2\pi f_c t) \cos(2\pi f_m t) + (1-2a) \sin(2\pi f_c t) \sin(2\pi f_m t) \right] \]

\[ \therefore m_1(t) = \frac{A_m}{2} \cos(2\pi f_m t) \]

\[ m_2(t) = \frac{A_m}{2} (1-2a) \sin(2\pi f_m t) \]

b) Let:

\[ s(t) = \frac{1}{2} A_m m(t) \cos(2\pi f_c t) + \frac{1}{2} A_m m(t) \sin(2\pi f_c t) \]

By adding the carrier frequency:

\[ s(t) = A_c [1 + \frac{1}{2} k_m m(t)] \cos(2\pi f_c t) + \frac{1}{2} k_m A_c m(t) \sin(2\pi f_c t) \]

where \( k_m \) is the percentage modulation.

After passing the signal through an envelope detector, the output will be:

\[ |s(t)| = A_c \left\{ \left[ 1 + \frac{1}{2} k_m m(t) \right]^2 + \left[ \frac{1}{2} k_m m(t) \right]^2 \right\}^{\frac{1}{2}} \]

\[ = A_c \left[ 1 + \frac{1}{2} k_m m(t) \right] \left\{ 1 + \left[ \frac{1}{2} k_m m(t) \left[ \frac{1}{2} k_m m(t) + 1 \right] \right]^2 \right\}^{\frac{1}{2}} \]

The second factor in \(|s(t)|\) is the distortion term \(d(t)\). For the example in (a), this becomes:

\[ d(t) = \left\{ 1 + \frac{(1-2a) \sin(2\pi f_m t)}{1 + \frac{1}{2} \cos(2\pi f_m t)} \right\}^{\frac{1}{2}} \]

c) Ideally, \(d(t)\) is equal to one. However, the distortion factor increases with decreasing \(a\). Therefore, the worst case exists when \(a = 0\).
Problem 3.19

(a,b) The spectrum of the message signal is illustrated below:

Correspondingly, the output of the upper first product modulator has the following spectrum:

The output of the lower first product modulator has the spectrum:

The output of the upper low pass filter has the spectrum
The output of the lower low pass filter has the spectrum:

\[
\begin{array}{c}
\frac{1}{2} M_1(f - f_0) \\
0 \rightarrow f_0 - f_0 \\
\frac{1}{2} M_1(f + f_0) \\
f_0 + f_0
\end{array}
\]

The output of the upper second product modulator has the spectrum:

\[
\begin{array}{c}
\frac{1}{4} M_2(f - f_0 + f_c) \\
\frac{1}{4} M_2(f - f_0 - f_c) \\
\frac{1}{4} M_2(f + f_0 + f_c) \\
\frac{1}{4} M_2(f + f_0 - f_c)
\end{array}
\]

The output of the lower second product modulator has the spectrum:

\[
\begin{array}{c}
\frac{1}{4} M_2(f - f_0 + f_c) \\
\frac{1}{4} M_2(f - f_0 - f_c) \\
\frac{1}{4} M_2(f + f_0 + f_c) \\
\frac{1}{4} M_2(f + f_0 - f_c)
\end{array}
\]

Adding the two second product modulator outputs, their upper sidebands add constructively while their lower sidebands cancel each other.

(c) To modify the modulator to transmit only the lower sideband, a single sign change is required in one of the channels. For example, the lower first product modulator could multiply the message signal by \(-\sin(2\pi f_c t)\). Then, the upper sideband would be cancelled and the lower one transmitted.
Problem 3.21

(a) The first product modulator output is

\[ v_1(t) = m(t) \cos(2\pi f_c t) \]

The second product modulator output is

\[ v_2(t) = v_2(t) \cos(2\pi (f_c + f_b) t) \]

The amplitude spectra of \( m(t) \), \( v_1(t) \), \( v_2(t) \), \( v_3(t) \) and \( s(t) \) are illustrated on the next page. We may express the voice signal \( m(t) \) as

\[ m(t) = \frac{1}{2} [m_+(t) + m_-(t)] \]

where \( m_+(t) \) is the pre-envelope of \( m(t) \), and \( m_-(t) = m_+(t)^* \) is its complex conjugate. The Fourier transforms of \( m_+(t) \) and \( m_-(t) \) are defined by (See Appendix 2)

\[ M_+(f) = \begin{cases} 2M(f), & f > 0 \\ 0, & f < 0 \end{cases} \]

\[ M_-(f) = \begin{cases} 0, & f > 0 \\ 2M(f), & f < 0 \end{cases} \]

Comparing the spectrum of \( s(t) \) with that of \( m(t) \), we see that \( s(t) \) may be expressed in terms of \( m_+(t) \) and \( m_-(t) \) as follows:

\[ s(t) = \frac{1}{8} m_+(t) \exp(-j2\pi f_b t) + \frac{1}{8} m_-(t) \exp(j2\pi f_b t) \]

\[ = \frac{1}{8} [m(t) + j\dot{m}(t)] \exp(-j2\pi f_b t) + \frac{1}{8} [m(t) - j\dot{m}(t)] \exp(j2\pi f_b t) \]

\[ = \frac{1}{4} m(t) \cos(2\pi f_b t) + \frac{1}{4} \dot{m}(t) \sin(2\pi f_b t) \]

(b) With \( s(t) \) as input, the first product modulator output is

\[ v_1(t) = s(t) \cos(2\pi f_c t) \]