Channel Estimation
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Introduction to Channel Estimation
Introduction

- Complex channel estimation (i.e., estimation of channel gain, which includes phase and amplitude) performed for each individual RAKE fingers is required for coherent detection (Maximal Ratio Combining).
- Complex channel estimation is performed with the assistance of known transmitted pilot symbols.
- The accuracy of the channel estimation is crucial for RAKE receiver performance, and it depends on the pilot channel energy, the channel estimation algorithms, and the environment conditions.
- In particular, mobile speed is required for a variety of channel estimation algorithms.
Pilot Symbol Transmission

The pilot symbols can be transmitted in two basic ways:

- In the case of *dedicate pilot channel scheme*, system has one physical channel fully dedicated to pilot symbol transmission.
  - E.g. Common Pilot Channel, CPICH, in downlink of WCDMA.
- Another option is to *insert pilot symbols into the data stream* (time multiplexed pilot symbols).
  - E.g. DPDCH/DPCCH in uplink of WCDMA.
One possible phase estimation architecture based on a dedicate pilot channel is shown in the following figure:

The output of the channel estimation is filtered by a low pass filter (LPF), whose bandwidth should be made adjustable to the Doppler frequency.
Time Multiplexed Pilot Symbols

- In the case of inserted pilot symbols, there are two well-known channel estimation techniques:
  - Decision feedback
  - Interpolation

- Decision feedback scheme
  - In this approach, hard decisions (decision of $\pm 1$) are made for the non-pilot (data) symbols.
    - The hard decisions may be made with the help of pilot symbols.
  - The data symbols are feedback, together with the pilot symbols, to form a continuous “pilot symbols” for channel estimation.
  - The output of channel is again filtered by a LPF which should be made adaptive to the Doppler frequency.
Time Multiplexed Pilot Symbols

- If hard decision results in small error rate, decision feedback schemes typically lead to better performance.

- Interpolation
  - In this scheme, since data symbols and pilot symbols are time multiplexed, channel estimations are first obtained for the pilot symbols and are then interpolated to obtain the channel gain of data symbols for coherent combining.
  - If the outputs of hard decision results in a high data error rate, which might due to small signal energy, severe fading effect, or high interference level, the decision feedback performance starts to degrade. Under such scenario, interpolation scheme might perform better since it uses only the pilot symbol energy for estimation.
Generic Pilot Based Channel Estimator
In making channel estimation, we assume that channel condition is constant over some time interval.

The complex channel estimation can be obtained by multiplying the “recovered pilot symbols” by the known pilot pattern.

In uplink direction, channel estimation is made with the help of pilot symbols within the DPCCH.

- Channel estimation is typically based on simple block averaging.
- Interpolation is typically performed between pilot blocks to estimate the channel status.

In downlink direction, channel estimation is made using the dedicated pilot channel, called Common Pilot Channel (CPICH).
Uplink Channel Estimator

Variables are defined as follows:

- $P$: Pilot symbols ($\pm 1$).
- $N_p$: Number of Pilot symbols.
- $D_d$: DPDCH symbols at UE transmitter.
- $D_c$: DPCCH symbols at UE transmitter.
- $S_I + jS_Q$: Complex scrambling code.
- $C_d$: DPDCH channelization code.
- $C_c$: DPCCH channelization code.
- $h_I + jh_Q$: Impulse response of complex fading channel.
- $R_d$: Received DPDCH data symbol at BS.
- $R_c$: Received DPCCH control symbol at BS.
At UE, data is BPSK modulated. The spread symbols at the UE transmitter are:

\[ D_d \cdot C_d + jD_c \cdot C_c \equiv D_I + jD_Q \]

After scrambling:

\[
\left( D_I + jD_Q \right) \cdot \left( S_I + jS_Q \right) = \left( D_I \cdot S_I - D_Q \cdot S_Q \right) + j \left( D_I \cdot S_Q + D_Q \cdot S_I \right)
\]

At base station, the received baseband signal is:

\[
\left[ \left( D_I \cdot S_I - D_Q \cdot S_Q \right) + j \left( D_I \cdot S_Q + D_Q \cdot S_I \right) \right] \cdot \left[ h_I + jh_Q \right]
\]

At base station, after descrambling:

\[
\frac{1}{2} \cdot \left( S_I - jS_Q \right) \cdot \left[ \left( D_I \cdot S_I - D_Q \cdot S_Q \right) + j \left( D_I \cdot S_Q + D_Q \cdot S_I \right) \right] \cdot \left[ h_I + jh_Q \right]
\]

\[
= \left( D_I \cdot h_I - D_Q \cdot h_Q \right) + j \left( D_I \cdot h_Q + D_Q \cdot h_I \right) = \left( D_I + jD_Q \right) \cdot \left( h_I + jh_Q \right)
\]
After despreading, we obtain:

**DPDCH:**  \( R_d = D_d \cdot (h_I + jh_Q) \)

**DPCCH:**  \( R_c = D_c \cdot (-h_Q + jh_I) \)

The channel condition information is estimated from the received pilot symbols:

\[
\hat{h}_I = \frac{1}{N_p} \cdot \sum_{N_p} \text{Im}\{R_c\} \cdot P
\]

\[
\hat{h}_Q = \frac{-1}{N_p} \cdot \sum_{N_p} \text{Re}\{R_c\} \cdot P
\]

where average is performed over \( N_p \) pilot symbols.
Uplink Channel Estimator

After interpolating the channel condition information for the DPCCH and DPDCH, channel compensation is performed by:

\[ \hat{h}_d \equiv \hat{h}_I + j\hat{h}_Q \]

\[ \hat{D}_d = \text{Re}\left\{ R_d \cdot \hat{h}_d^* \right\} = \text{Re}\left\{ R_d \cdot \left( \hat{h}_I - j\hat{h}_Q \right) \right\} \]

\[ = \text{Re}\left\{ R_d \right\} \cdot \hat{h}_I + \text{Im}\left\{ R_d \right\} \cdot \hat{h}_Q \]

Estimated DPDCH = \[ D_d \cdot \left( h_I \cdot \hat{h}_I + h_Q \cdot \hat{h}_Q \right) \]

\[ \hat{h}_c \equiv -\hat{h}_Q + j\hat{h}_I \]

\[ \hat{D}_c = \text{Re}\left\{ R_c \cdot \hat{h}_c^* \right\} = \text{Re}\left\{ R_c \cdot \left( -\hat{h}_Q - j\hat{h}_I \right) \right\} \]

\[ = -\text{Re}\left\{ R_c \right\} \cdot \hat{h}_Q + \text{Im}\left\{ R_c \right\} \cdot \hat{h}_I \]

Estimated DPCCH = \[ D_c \cdot \left( h_Q \cdot \hat{h}_Q + h_I \cdot \hat{h}_I \right) \]
Downlink Channel Estimator

- Variables are defined as follows:
  \[ P_i + jP_q \]: Complex Pilot symbols at BS \( P_i = P_q = P \).

- \( N_p \): Number of Pilot symbols (10 Symbols/Slot).

- \( D_i + jD_q \): Complex DPCH symbols at BS transmitter.

- \( S_I + jS_Q \): Complex scrambling code.

- \( C_I + jC_Q \): Complex channelization code \( C_I = C_Q \).

- \( h_I + jh_Q \): Impulse response of complex fading channel.

- \( R_d \): Received DPCH data symbol at UE.

- \( R_{PI} = R_P + jR_P \): Received CPICH symbols at UE.

- Note that the I and Q branches of either DPCH or CPICH are spread to the chip rate by the same channelization code.
At BS, data is QPSK modulated. The spread symbols at the BS transmitter are:

\[ D_i + jD_Q \equiv (D_i \cdot C_i) + j(D_q \cdot C_Q) \]

After scrambling:

\[ (D_i + jD_Q) \cdot (S_i + jS_Q) = (D_i \cdot S_i - D_Q \cdot S_Q) + j(D_i \cdot S_Q + D_Q \cdot S_i) \]

At UE, the received baseband signal is:

\[ \left[ (D_i \cdot S_i - D_Q \cdot S_Q) + j(D_i \cdot S_Q + D_Q \cdot S_i) \right] \cdot \left[ h_i + jh_Q \right] \]

At UE, after descrambling:

\[ \frac{1}{2} \cdot (S_i - jS_Q) \cdot \left[ (D_i \cdot S_i - D_Q \cdot S_Q) + j(D_i \cdot S_Q + D_Q \cdot S_i) \right] \cdot \left[ h_i + jh_Q \right] \]

\[ = (D_i \cdot h_i - D_Q \cdot h_Q) + j(D_i \cdot h_Q + D_Q \cdot h_i) = (D_i + jD_Q) \cdot (h_i + jh_Q) \]
Downlink Channel Estimator

After despreading, we obtain (note that $C_I = C_Q$):

**DPCH:**

$$R_d = \left( D_i \cdot h_I - D_q \cdot h_Q \right) + j \left( D_i \cdot h_Q + D_q \cdot h_I \right)$$

$$= \left( D_i + jD_q \right) \cdot \left( h_I + jh_Q \right)$$

**CPICH:**

$$R_{PI} = R_P \left( h_I - h_Q \right) + jR_P \left( h_Q + h_I \right)$$

The channel condition information is estimated from the received pilot symbols:

$$\hat{h}_I = \frac{1}{2 N_P} \cdot \sum_{N_p} \left\{ \text{Re} \left\{ R_{PI} \right\} \cdot P + \text{Im} \left\{ R_{PI} \right\} \cdot P \right\}$$

$$\hat{h}_Q = \frac{1}{2 N_P} \cdot \sum_{N_p} \left\{ \text{Im} \left\{ R_{PI} \right\} \cdot P - \text{Re} \left\{ R_{PI} \right\} \cdot P \right\}$$

where average is performed over $N_p$ pilot symbols.
After obtaining the channel condition information for the DPCH, channel compensation for the I branch is performed by:

\[ \hat{h}_d \equiv \hat{h}_I + j \hat{h}_Q \]

\[
\hat{D}_i = \text{Re}\left\{ R_d \cdot \hat{h}_d^* \right\} = \text{Re}\left\{ R_d \cdot \left( \hat{h}_I - j \hat{h}_Q \right) \right\}
\]

\[ = \text{Re}\left\{ R_d \right\} \cdot \hat{h}_I + \text{Im}\left\{ R_d \right\} \cdot \hat{h}_Q \]

Estimated DPCH\(_I\) = \left( D_i \cdot h_I - D_q \cdot h_Q \right) \cdot \hat{h}_I + \left( D_i \cdot h_Q + D_q \cdot h_I \right) \cdot \hat{h}_Q
\]

\[ = D_i \cdot \left( h_I \cdot \hat{h}_I + h_Q \cdot \hat{h}_Q \right) + D_q \cdot \left( h_I \cdot \hat{h}_Q - h_Q \cdot \hat{h}_I \right) \]

For perfect channel estimation \( \left( \hat{h}_I = h_I, \hat{h}_Q = h_Q \right) \):

Estimated DPCH\(_I\) \approx D_i \cdot \left( h_I^2 + h_Q^2 \right)
After obtaining the channel condition information for the DPCH, channel compensation for the Q branch is performed by:

\[ \hat{h}_d \equiv \hat{h}_I + j\hat{h}_Q \]

\[ \hat{D}_q = \text{Im}\left\{ R_d \cdot \hat{h}_d^* \right\} = \text{Im}\left\{ R_d \cdot \left( \hat{h}_I - j\hat{h}_Q \right) \right\} \]

\[ = \text{Im}\left\{ R_d \right\} \cdot \hat{h}_I - \text{Re}\left\{ R_d \right\} \cdot \hat{h}_Q \]

Estimated DPCH \( Q \)

\[ = \left( D_i \cdot h_Q + D_q \cdot h_I \right) \cdot \hat{h}_I - \left( D_i \cdot h_I - D_q \cdot h_Q \right) \cdot \hat{h}_Q \]

\[ = D_q \cdot \left( h_I \cdot \hat{h}_I + h_Q \cdot \hat{h}_Q \right) + D_i \cdot \left( h_Q \cdot \hat{h}_I - h_I \cdot \hat{h}_Q \right) \]

For perfect channel estimation \( \left( \hat{h}_I = h_I, \hat{h}_Q = h_Q \right) \):

Estimated DPCH \( Q \) \( \approx D_q \cdot \left( h_I^2 + h_Q^2 \right) \)
WCDMA Channel Estimation Techniques
Channel Estimation Techniques

- Numerous approaches can be used for pilot aided channel estimation.
- We need to take into account at least the following factors:
  - The performance of the algorithms.
  - The complexity relative to the current DSP processing power.
- In WCDMA, channel estimation typically consists of two steps:
  - Periodic channel sampling by virtue of known pilot symbols: the instantaneous channel estimate in each time slot is obtained by averaging the channel estimates calculated from each pilot symbol.
  - Channel tracking (interpolation): the tracking of the time-varying fading channel is realized through a filter to obtain the channel estimates from the data symbols.
In general, the channel estimate from each pilot symbol is provided by multiplying the received pilot symbols $r_l(n,k)$ by the complex conjugate of the known pilot symbols:

$$\tilde{h}_l = r_l(n,k) \cdot p^*(n,k) \quad n = 1, 2, \ldots, N_p$$

where $l$ is the multi-path index, $N_p$ is the number of pilot symbols and time slot index is $k$.

To minimize the effects of noise and interference, an average is taken over the number of pilot symbols within a time slot:

$$\bar{h}_l(k) = \frac{1}{N_p} \sum_{n=1}^{N_p} \tilde{h}_l(n,k)$$
Simple Average

- This is a simple but effective method to estimate the channel complex gain.
- Simple average technique averages the received pilot symbols within one time slot.
- Simple average technique reduces the sampling rate from symbol rate to slot rate.

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DPCCH Symbols ───> Pilot Field Timing ┼ Slot Timing ┼ Pilot Pattern
                   ⨌                     ⨌                     ⨌
                                      | Channel Estimate
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Weighted Multi Slot Average (WMSA)

Weighted-Multislot Averaging

First, instantaneous channel estimation is performed by using the $N_p$ pilot symbols belonging to the $n$th slot:

$$\tilde{h}_l(n) = \frac{1}{N_p} \sum_{m=0}^{N_p-1} r_l(n,m)$$

The channel estimate to be used for the $n$-th slot data is obtained by using $2K$ consecutive channel estimates:

$$\overline{h}_l = \sum_{i=-K+1}^{K} \alpha(i) \cdot \tilde{h}_l(n+i)$$

where $\alpha(i) \leq 1$ is the weighting factor.

By properly choosing weighting factors, $\alpha(i)$, accurate channel estimation is possible, particularly in slow fading environments.
The alpha tracker can be thought of as an IIR low-pass filter. The difference equation describing the alpha track is:

\[ y(n) = (1 - \alpha) \cdot x(n-1) + \alpha \cdot y(n-1) \]

The filter can be described in the z domain:

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{(1 - \alpha)z^{-1}}{1 - \alpha z^{-1}} \]
Substituting $z$ with $e^{j\omega}$, we can obtain the frequency response:

$$
|H(e^{j\omega})| = \frac{1 - \alpha}{\left(1 + \alpha^2 - 2\alpha \cos \omega\right)^{1/2}}
$$

$$
\angle H(e^{j\omega}) = \tan^{-1} \frac{\sin \omega}{\alpha - \cos \omega}
$$

The cut-off frequency $\omega_c$ of the IIR filter is given by:

$$
\omega_c = \cos^{-1} \frac{-\alpha^2 + 4\alpha - 1}{2\alpha}
$$
There are a variety of channel estimation techniques that can be applied:

- **Adaptive Filter Techniques**

- **Wiener Filter**

- **Forward Linear Prediction**
Interpolation
Newton’s Divided-Difference Formula

Assume values of function, $f(x)$, are known at suitably specialized points, $x_0, x_1, \cdots, x_n$, which need not be equally spaced or taken in consecutive order.

Newton’s divided-difference formula:

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + \cdots$$
$$+ (x-x_0)(x-x_1)\cdots(x-x_{n-1})f(x_0,x_1,\cdots,x_n)$$
$$+ (x-x_0)(x-x_1)\cdots(x-x_n)f(x,x_0,x_1,\cdots,x_n)$$

$$\equiv p_n(x) + r_{n+1}(x)$$

$$p_n(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + \cdots$$
$$+ (x-x_0)(x-x_1)\cdots(x-x_{n-1})f(x_0,x_1,\cdots,x_n)$$

$$r_{n+1}(x) = (x-x_0)(x-x_1)\cdots(x-x_n)f(x,x_0,x_1,\cdots,x_n) \equiv \text{error term}$$
Lagrange’s Interpolation Formula

Lagrange’s interpolation formula is a polynomial of degree $n$ (or less) since each term on the right hand side is a polynomial of degree $n$.

When $x=x_i$, $i=0,1,\cdots,n$, every fraction except the $(i+1)^{th}$ vanishes because of the factor $(x-x_i)$.

\[
f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_0) + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} f(x_n)
\]
Forward Gregory-Newton Interpolation Formula

- When the points \( x_0, x_1, \ldots, x_n \) on which Newton’s divided-difference formula is based are regularly spaced with tabular interval \( h \), it is generally more convenient to express Newton’s divided-difference formula in terms of ordinary differences.

- Let \( x=x_0+rh \) and \( x_k=x_0+kh \), we have \( x-x_k=h(r-k) \ k=0,1,\ldots,n \). Therefore, \( (x-x_0)(x-x_1)\cdots(x-x_j)=h^{j+1}r(r-1)\cdots(r-j) \).

- Using:

\[
f(x_0, x_1, \ldots, x_{j+1}) = \frac{\Delta^{j+1}f_0}{(j+1)!h^{j+1}}
\]

\[
\Delta f_k = f_{k+1} - f_k
\]

\[
\Delta^n f_k = \Delta^{n-1} f_{k+1} - \Delta^{n-1} f_k
\]

- We have, from Newton’s divided difference formula, Forward Gregory-Newton Interpolation formula:

\[
f(x) = f(x_0 + rh) = f_0 + r \Delta f_0 + \frac{r(r-1)}{2!} \Delta^2 f_0 + \frac{r(r-1)(r-2)}{3!} \Delta^3 f_0 + \ldots
\]
Backward Gregory-Newton Interpolation Formula

- Forward Gregory-Newton Interpolation formula is especially adapted to interpolation for smaller value of \( x \).
- Let \( x = x_0 + rh \) and \( x_k = x_0 - kh \), we have \( x - x_k = h(r+k) \) for \( k = 0,1,\ldots,n \). Therefore, \((x-x_0)(x-x_1)\cdots(x-x_j) = h^{j+1}r(r+1)\cdots(r+j)\).
- Using:

  \[
  f \left( x_0, x_1, \ldots, x_{j+1} \right) = \frac{\Delta^{j+1} f_{-j-1}}{(j+1)!h^{j+1}}
  \]

- We have, from Newton’s divided difference formula, the Backward Gregory-Newton Interpolation formula:

  \[
  f(x) \equiv f(x_0 + rh) = f_0 + r\Delta f_{-1} + \frac{r(r+1)}{2!} \Delta^2 f_{-2} + \frac{r(r+1)(r+2)}{3!} \Delta^3 f_{-3} + \cdots
  \]
Let \( x_0 = x_0, x_1 = x_0 + h, x_2 = x_0 - h, x_3 = x_0 + 2h, x_4 = x_0 - 2h, \ldots \)

Using:

\[
f(x_0, x_1, \ldots, x_{j+1}) = \frac{\Delta^{j+1} f_{-j-1}}{(j+1)!h^{j+1}}
\]

We have, from Newton’s divided difference formula, the Forward Newton-Gauss Interpolation formula:

\[
f(x) = f(x_0 + rh) = f_0 + r \Delta f_0 + \frac{r(r-1)}{2!} \Delta^2 f_{-1} + \frac{r(r-1)(r+1)}{3!} \Delta^3 f_{-1} + \frac{r(r-1)(r+1)(r-2)}{4!} \Delta^4 f_{-2} + \ldots
\]

\[
= f_0 + r \delta f_{1/2} + \frac{r(r-1)}{2!} \delta^2 f_0 + \frac{(r+1)r(r-1)}{3!} \delta^3 f_{1/2} + \frac{(r+1)r(r-1)(r-2)}{4!} \delta^4 f_0 + \ldots
\]

where \( \delta f_{k+1/2} = f_{k+1} - f_k \).
Backward Newton-Gauss Interpolation Formula

- Let $x_0 = x_0, x_1 = x_0 - h, x_2 = x_0 + h, x_3 = x_0 - 2h, x_4 = x_0 + 2h, \ldots$
- We have, from Newton’s divided difference formula, 
  **Backward Newton-Gauss Interpolation formula**:

$$f(x) \equiv f(x_0 + rh) = f_0 + r \delta f_{-1/2} + \frac{(r+1)r}{2!} \delta^2 f_0 + \frac{(r+1)r(r-1)}{3!} \delta^3 f_{-1/2} + \frac{(r+2)(r+1)r(r-1)}{4!} \delta^4 f_0 + \ldots$$

- We can also have Stirling’s interpolation formula:

$$f(x) \equiv f(x_0 + rh) = f_0 + \frac{r}{1!} \cdot \frac{\delta f_{1/2}}{2} + \frac{r^2}{2!} \delta^2 f_0 + \frac{r(r^2 - 1)}{3!} \cdot \frac{\delta^3 f_{1/2}}{2} + \frac{r^2(r^2 - 1)}{4!} \delta^4 f_0 + \ldots$$
Fast Channel Estimation
Fast Channel Estimation

- In order for the power control to be effective, the TPC symbol must be decoded with minimum possible delay to adjust the transmit power so that the overall power control delay should be one time slot.

- Similar comment also applies for the SIR measurement since SIR must be calculated and compared to a set point to determine the TPC bits.

- The only difference between the fast channel estimation and the ordinary (slow) channel estimation is whether interpolation is used or not.

- As a result, fast channel estimation is obtained by averaging the $N_p$ pilot symbols within a time slot.

$$\bar{h}_l(k) = \frac{1}{N_p} \sum_{n=1}^{N_p} \tilde{h}_l(n,k)$$