Synchronization Schemes

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Introduction

- In recent years, orthogonal frequency division multiplexing (OFDM) has become the most attractive transmission scheme for digital communications.

- However, OFDM systems are very sensitive to synchronization errors because these errors result in both the inter-symbol interference (ISI) and inter-carrier interference (ICI).

- Therefore, accurate timing and frequency offset estimation schemes are indispensable for OFDM systems.
Introduction

- All synchronization algorithms can be classified into two categories: data-aided and blind.
- If pilot symbol is used to help estimate timing offset and/or frequency offset, the corresponding synchronization schemes are called data-aided.
- On the other hand, if there is no pilot symbol, the corresponding synchronization schemes are considered to be blind.
- The estimation methods based on training symbols to jointly estimate timing offset and frequency offset are often employed in present communication standards, e.g. IEEE802.11a, and IEEE802.15.3a (MB-OFDM).
Introduction
Introduction
**Packet format for IEEE 802.11a.**

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RATE 4 bits</td>
<td>Reserved 1 bit</td>
</tr>
<tr>
<td>LENGTH 12 bits</td>
<td>Parity 1 bit</td>
</tr>
<tr>
<td>Tail 6 bits</td>
<td>SERVICE 16 bits</td>
</tr>
<tr>
<td>Pad Bits</td>
<td>PSDU</td>
</tr>
<tr>
<td></td>
<td>Tail 6 bits</td>
</tr>
<tr>
<td></td>
<td>Pad Bits</td>
</tr>
</tbody>
</table>

**PLCP Header**

- Coded/OFDM (BPSK, r=1/2)
- Coded/OFDM (RATE is indicated in SIGNAL)

**PLCP Preamble 12 Symbols**

- SIGNAL
  - One OFDM Symbol
- DATA
  - Variable Number of OFDM Symbols
Introduction

Task 1: Packet Detection

Task 2: Freq. & Timing Synchronization

Data Transmission
Synchronization of an OFDM signal requires finding the symbol timing and carrier frequency offset.

A rapid synchronization method is presented for an orthogonal frequency-division multiplexing (OFDM) system using either a continuous transmission or a burst operation over a frequency-selective channel.

The start of the frame and the beginning of the symbol can be found, and carrier frequency offsets of many subcarrier spacings can be corrected.

This paper describes a method to acquire synchronization for either a continuous stream of data as in a broadcast application or for bursty data as in a wireless local area network (WLAN).
The OFDM signal is generated at baseband by taking the inverse fast Fourier transform (IFFT) of quadrature amplitude modulated (QAM) or phase-shift keyed (PSK) subsymbols $c_k = a_k + jb_k$.

An OFDM symbol has a useful period $T$ and preceding each symbol is a cyclic prefix of length $T_g$, which is longer than the channel impulse response so that there will be no intersymbol interference (ISI).

The frequencies of the complex exponentials are $f_k = k/T$, and the useful part for $2N+1$ subcarriers is given by

$$u(t) = \sum_{k=-N}^{N} c_k \exp(j2\pi f_k t), \quad 0 \leq t \leq T.$$ 

A carrier frequency offset of $\Delta f$ causes a phase rotation of $2\pi \Delta ft$. 

Symbol Timing Estimation Algorithm

- The symbol timing recovery relies on searching for a training symbol with two identical halves in the time domain, which will remain identical after passing through the channel, except that there will be a phase difference between them caused by the carrier frequency offset.
- The two halves of the training symbol are made identical (in time order) by transmitting a pseudonoise (PN) sequence on the even frequencies, while zeros are used on the odd frequencies.
- This means that at each even frequency one of the points of a QPSK constellation is transmitted.
Symbol Timing Estimation Algorithm

The second training symbol contains a PN sequence on the odd frequencies to measure these subcarriers, and another PN sequence on the even frequencies to help determine frequency offset.

<table>
<thead>
<tr>
<th>freq. num. k</th>
<th>$c_{1,k}$</th>
<th>$c_{2,k}$</th>
<th>$u_k = \sqrt{2} \frac{c_{2,k}}{c_{1,k}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>7+7j</td>
<td>5-5j</td>
<td>-j</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
<td>-5-5j</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>-7+7j</td>
<td>-5-5j</td>
<td>j</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>-5+5j</td>
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<td>7+7j</td>
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<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>5+5j</td>
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<td>2</td>
<td>7-7j</td>
<td>-5+5j</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>5-5j</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7+7j</td>
<td>5+5j</td>
<td>1</td>
</tr>
</tbody>
</table>
Symbol Timing Estimation Algorithm

- After sampling, the complex samples are denoted as $r_m$.
- Consider the first training symbol where the first half is identical to the second half (in time order), except for a phase shift caused by the carrier frequency offset.
- If the conjugate of a sample from the first half is multiplied by the corresponding sample from the second half ($T/2$ seconds later), the effect of the channel should cancel, and the result will have a phase of approximately $\phi = 2\pi \Delta f T = \pi T \Delta f$.
- At the start of the frame, the products of each of these pairs of samples will have approximately the same phase, so the magnitude of the sum will be a large value.
Symbol Timing Estimation Algorithm

- Let there be $L$ complex samples in one-half of the first training symbol (excluding the cyclic prefix), and let the sum of the pairs of products be $P(d) = \sum_{m=0}^{L-1} (r_{d+m}^* r_{d+m+L})$ which can be implemented with the iterative formula $P(d+1) = P(d) + (r_{d+L}^* r_{d+2L}) - (r_{d}^* r_{d+L})$.

- Note that $d$ is a time index corresponding to the first sample in $2L$ a window of samples.

- The received energy for the second half-symbol is defined by $R(d) = \sum_{0}^{L-1} |r_{d+m+L}|^2$.

- A timing metric can be defined as $M(d) = |P(d)|^2 / (R(d))^2$. 
Fig. 3 shows an example of the timing metric as a window slides past coincidence for the AWGN channel for an OFDM signal with 1000 subcarriers, a carrier frequency offset of 12.4 subcarrier spacings, and an signal-to-noise ratio (SNR) of 10 dB, where the SNR is the total signal (all the subcarriers) to noise power ratio.
Symbol Timing Estimation Algorithm

```
\begin{align*}
r(0) & \quad r(1) & \quad \cdots & \quad r(n-2) & \quad r(n-1) & \quad r(0) & \quad r(1) & \quad \cdots & \quad r(n-2) & \quad r(n-1) \\
\end{align*}
\text{Length} = \frac{N}{2}
```

```
\begin{align*}
r(n-1) & \quad r(0) & \quad r(1) & \quad \cdots & \quad r(n-2) & \quad r(n-1) & \quad r(0) & \quad r(1) & \quad \cdots & \quad r(n-2) & \quad r(n-1) \\
\end{align*}
\text{Sliding window (length = } N\text{)}
```
Two methods to determine the symbol timing are compared on the basis of reduction in SNIR.

The first method is to simply find the maximum of the timing metric.

The second method is to find the maximum, find the points to the left and right in the time domain, which are 90% of the maximum, and average these two 90% times to find the symbol timing estimate.

The rationale behind this method is that the best timing points typically lie in a plateau.
Symbol Timing Estimation Algorithm
Carrier Frequency Offset Estimation Algorithm

The main difference between the two halves of the first training symbol will be a phase difference of \( \phi = \pi T \Delta f \) which can be estimated by \( \phi = \angle(P(d)) \) near the best timing point.

If \( |\phi| \) can be guaranteed to be less than \( \pi \), then the frequency offset estimate is \( \Delta f = \hat{\phi} / (\pi T) \) and the even PN frequencies on the second training symbol would not be needed.

Otherwise, the actual frequency offset would be \( \frac{\phi}{\pi T} + \frac{2z}{T} \) where \( z \) is an integer.

By partially correcting the frequency offset, adjacent carrier interference (ACI) can be avoided, and then the remaining offset of \( 2z/T \) can be found.
After the two training symbols are frequency corrected, let their FFT’s be $x_{1,k}$ and $x_{2,k}$, and let the differentially-modulated PN sequence on the even frequencies of the second training symbol be $v_k$ (as illustrated in Table I).

The PN sequence $v_k$ will appear at the output except it will be shifted by $2z$ positions because of the uncompensated frequency shift of $2z/T$.

Let be the set of indices for the even frequency components, $X=\{-W,-W+2,\ldots,-4,-2,2,4,\ldots,W-2,W\}$. 

The number of even positions shifted can be calculated by finding $\hat{g}$ to maximize

$$B(g) = \sum_{k \in X} x_{1,k+2g}^* v_k^* x_{2,k+2g}^2 / \left( 2 \left( \sum_{k \in X} |x_{2,k}|^2 \right)^2 \right)$$

with integer $g$ spanning the range of possible frequency offsets and $W$ being the number of even frequencies with the PN sequence.

Then the frequency offset estimate would be $\hat{\Delta f} = \hat{\phi} / (\pi T) + 2\hat{g} / T$. 
We present and evaluate the joint maximum likelihood (ML) estimation of the time and carrier-frequency offset in OFDM systems.

Our novel algorithm exploits the cyclic prefix preceding the OFDM symbols, thus reducing the need for pilots.

Simulations show that the frequency estimator may be used in a tracking mode and the time estimator in an acquisition mode.
System Model

Fig. 1. OFDM system, transmitting subsequent blocks of $N$ complex data.
An accepted means of avoiding intersymbol interference (ISI) and preserving orthogonality between subcarriers is to copy the last \( L \) samples of the body of the OFDM symbol (\( N \) samples long) and append them as a preamble—the cyclic prefix—to form the complete OFDM symbol.

The effective length of the OFDM symbol as transmitted is this cyclic prefix plus the body (\( L+N \) samples long).

In the following analysis, we assume that the channel is nondispersive and that the transmitted signal \( s(k) \) is only affected by complex additive white Gaussian noise (AWGN) \( n(k) \).
System Model

Consider two uncertainties in the receiver of this OFDM symbol: the uncertainty in the arrival time of the OFDM symbol and the uncertainty in carrier frequency.

The first uncertainty is modeled as a delay in the channel impulse response $\delta(k-\theta)$, where $\theta$ is the integer-valued unknown arrival time of a symbol.

The latter is modeled as a complex multiplicative distortion of the received data in the time domain $\exp(j2\pi k/N)$, where $\varepsilon$ denotes the difference in the transmitter and receiver oscillators as a fraction of the intercarrier spacing ($1/N$ in normalized frequency).

The FO result in a phase shift in time domain can be written as:

$$\theta = 2\pi \Delta ft = 2\pi \Delta f \frac{T}{N} k = 2\pi \frac{\Delta f}{1/T} \frac{k}{N} = 2\pi \varepsilon k/N$$
System Model

Notice that all subcarriers experience the same shift \( \mathcal{E} \).

These two uncertainties and the AWGN thus yield the received signal \( r(k) = s(k - \theta) e^{j2\pi nk/N} + n(k) \).

Two other synchronization parameters are not accounted for in this model.

First, an offset in the carrier phase may affect the symbol error rate in coherent modulation, however, this can be eliminated by differentially encoded.

An offset in the sampling frequency will also affect the system performance, but we assume that such an offset is negligible.
Now, consider the transmitted signal $s(k)$. This is the DFT of the data symbols $x_k$, which we assume are independent.

Hence, $s(k)$ is a linear combination of independent, identically distributed random variables. If the number of subcarriers is sufficiently large, we know from the central limit theorem that $s(k)$ approximates a complex Gaussian process whose real and imaginary parts are independent.

This process, however, is not white since the appearance of a cyclic prefix yields a correlation between some pairs of samples that are spaced $N$ samples apart.

Hence, $r(k)$ is not a white process either, but because of its probabilistic structure, it contains information about the time offset $\theta$ and carrier frequency offset $\varepsilon$. 
ML Estimation

Assume that we observe $2N+L$ consecutive samples of $r(k)$, cf. Fig. 2, and that these samples contain one complete $(N+L)$-sample OFDM symbol.
ML Estimation

- Define the index sets $I \equiv \{\theta, \ldots, \theta + L - 1\}$ and $I' \equiv \{\theta + N, \ldots, \theta + N + L - 1\}$.
- Collect the observed samples in the $(2N + L) \times 1$-vector
  $\mathbf{r} \equiv [r(1) \ldots r(2N + L)]^T$.
- Notice that the samples in the cyclic prefix and their copies $r(k)$, $k \in I \cup I'$ are pairwise correlated, i.e.,
  $$\forall k \in I: \ E\{r(k)r^*(k + m)\} = \begin{cases} 
\sigma_s^2 + \sigma_n^2 & m = 0 \\
\sigma_s^2 e^{-j2\pi \epsilon} & m = N \\
0 & \text{otherwise}
\end{cases}$$
- while the remaining samples $r(k)$, $k \notin I \cup I'$ are mutually uncorrelated.
Using the correlation properties of the observations \( \mathbf{r} \), the log-likelihood function can be written as

\[
\Lambda(\theta, \varepsilon) = \log f(\mathbf{r} | \theta, \varepsilon) = \log \left[ f(r(1), r(2), \ldots, r(2N + L)) \right]
\]

\[
= \log \left\{ \prod_{k \in \mathbb{I}} f(r(k), r(k + N)) \prod_{k \notin \mathbb{I}} f(r(k)) \right\}
\]

\[
= \log \left\{ \prod_{k \in \mathbb{I}} \frac{f(r(k), r(k + N))}{f(r(k)) f(r(k + N))} \prod_{k} f(r(k)) \right\}
\]

Under the assumption that \( \mathbf{r} \) is a jointly Gaussian vector and omit some factor, we can show that:

\[
\Lambda(\theta, \varepsilon) = |r(\theta)| \cos(2\pi \varepsilon + \angle r(\theta)) - \rho \Phi(\theta) \tag{1}
\]
ML Estimation

The joint complex gaussian PDF can be expressed as

\[ f_U(u) = \frac{1}{(2\pi)^N \det(\Lambda)} \exp(-\frac{1}{2} u^H \Lambda^{-1} u) \]

where \( u = [u(1), u(2), \ldots, u(N)] \) and \( \Lambda = \frac{1}{2} E[uu^H] \).

Note that utilize \( AB^* + A^*B = 2\text{Re}[AB^*] \), we can easily show

\[
\exp\left\{ \frac{|r(k)|^2 - 2\rho \text{Re}\{e^{j2\pi\epsilon} r(k)r^*(k+N)\} + |r(k+N)|^2}{(\sigma_s^2 + \sigma_n^2)(1 - \rho^2)} \right\}
\]

\[ f(r(k), r(k+N)) = \frac{1}{\pi^2 (\sigma_s^2 + \sigma_n^2)^2 (1 - \rho^2)} \sum_{k=m}^{m+L-1} |r(k+N)|^2 + |r(k)|^2 \]

where \( \rho = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} = \frac{\text{SNR}}{1 + \text{SNR}} \), \( \gamma(m) = \sum_{k=m}^{m+L-1} r(k)r^*(k+N) \), \( \Phi(m) = \sum_{k=m}^{m+L-1} |r(k+N)|^2 + |r(k)|^2 \)

and \( \Lambda(\theta, \epsilon) = c_1 + c_2 \left[ |\gamma(\theta)| \cos(2 \pi \epsilon + \angle \gamma(\theta)) - \rho \Phi(\theta) \right] \).
The maximization of the log-likelihood function can be performed in two steps: \( \max_{(\theta, \varepsilon)} \Lambda(\theta, \varepsilon) = \max_{\theta} \max_{\varepsilon} \Lambda(\theta, \varepsilon) = \max_{\theta} \Lambda(\theta, \hat{\varepsilon}_{ML}(\theta)) \).

The maximum with respect to the frequency offset is obtained when the cosine term in (1) equals one.

This yields the ML estimation of \( \varepsilon \) which is \( \hat{\varepsilon}_{ML}(\theta) = \frac{-1}{2\pi} \angle \gamma(\theta) + n \).

We assume that an acquisition, or rough estimate, of the frequency offset has been performed and that \( \varepsilon < |0.5| \); thus, \( n=0 \).

Since the cosine term equals to one, the log-likelihood function of \( \theta \) becomes \( \Lambda(\theta, \hat{\varepsilon}_{ML}(\theta)) = |\gamma(\theta)| - \rho \Phi(\theta) \) and the joint ML estimation of \( \theta \) and \( \varepsilon \) becomes \( \hat{\theta}_{ML} = \arg \max \{|\gamma(\theta)| - \rho \Phi(\theta)| \} \)

\( \hat{\varepsilon}_{ML}(\theta) = \frac{-1}{2\pi} \angle \gamma(\hat{\theta}_{ML}) \)
Fig. 5. Performance of the time (top) and frequency (bottom) estimators for the AWGN channel (4, 10, and 16 dB). The dimensionless performance measure is expressed in squared units relative to the sample interval (top) and the inter-tone spacing (bottom). The number of subcarriers is $N = 256$.

Fig. 6. Performance of the time (top) and frequency (bottom) estimators for the AWGN channel ($L = 4$, $L = 8$, and $L = 15$) and the dispersive channel ($L = 15$). The dimensionless performance measure is expressed in squared units relative to the sample interval (top) and the inter-tone spacing (bottom). The number of subcarriers is $N = 256$. 
 Derived of ML Estimator

\[ f(u) = \frac{1}{(2\pi)^N \det(\Lambda)} \exp \left( \frac{-1}{2} u^H \Lambda^{-1} u \right) \]

where \( u = [u(1) \ u(2) \ \cdots \ u(N)]^T \) and \( \Lambda = 0.5E[uu^H] \) is the correlation matrix.

thus \( f(u) = \frac{1}{\pi \sigma^2} \exp \left( \frac{-|u|^2}{\sigma^2} \right), \quad \sigma^2 = \text{var}(u), \)

and \( f(r(k), r(k+N)) = \frac{1}{4\pi^2 \det(\Lambda)} \exp \left( \frac{-1}{2} u^H \Lambda^{-1} u \right) \)

\[ \Lambda = 0.5E \begin{bmatrix} r(k) \\ r(k+N) \end{bmatrix} \begin{bmatrix} r^*(k) & r^*(k+N) \end{bmatrix} = 0.5 \begin{bmatrix} \sigma_s^2 + \sigma_n^2 & \sigma_s e^{-j2\pi e} \\ \sigma_s e^{j2\pi e} & \sigma_s^2 + \sigma_n^2 \end{bmatrix} \]

define: \( \rho = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} \), then we have: \( \rho^2 = \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2} \), \( 1 - \rho^2 = \frac{2\sigma_s^2 \sigma_n^2 + \sigma_n^4}{(\sigma_s^2 + \sigma_n^2)^2} \)

\[ \begin{equation} \begin{aligned} \det(\Lambda) &= \frac{(\sigma_s^2 + \sigma_n^2)^2 - \sigma_s^4}{4} = \frac{(\sigma_s^2 + \sigma_n^2)^2}{4} (1 - \rho^2) \end{aligned} \end{equation} \]
Derived of ML Estimator

\[ -0.5u^H \Lambda^{-1} u = - \begin{bmatrix} r^*(k) & r^*(k+N) \end{bmatrix} \begin{bmatrix} \sigma_s^2 + \sigma_n^2 & -\sigma_s^2 e^{-j2\pi\epsilon} \\ -\sigma_s^2 e^{j2\pi\epsilon} & \sigma_s^2 + \sigma_n^2 \end{bmatrix} \begin{bmatrix} r(k) \\ r(k+N) \end{bmatrix} \sqrt{ \left( \sigma_s^2 + \sigma_n^2 \right)^2 (1 - \rho^2) } \]

\[ = - \frac{\left| r(k) \right|^2 \left( \sigma_s^2 + \sigma_n^2 \right) - r(k)r^*(k+N)\sigma_s^2 e^{j2\pi\epsilon} - r(k)r^*(k+N)\sigma_s^2 e^{-j2\pi\epsilon} + |r(k+N)|^2 \left( \sigma_s^2 + \sigma_n^2 \right)}{\left( \sigma_s^2 + \sigma_n^2 \right)^2 (1 - \rho^2)} \]

\[ = - \frac{\left| r(k) \right|^2 \rho e^{j2\pi\epsilon} \left[ r(k)r^*(k+N) - r(k)r^*(k+N) \rho e^{-j2\pi\epsilon} + |r(k+N)|^2 \right]}{\left( \sigma_s^2 + \sigma_n^2 \right)(1 - \rho^2)} \]

\[ = - \left[ \left| r(k) \right|^2 - 2 \text{Re} \left[ \rho \cdot r(k)r^*(k+N)e^{j2\pi\epsilon} \right] + |r(k+N)|^2 \right] \sqrt{ \left( \sigma_s^2 + \sigma_n^2 \right)(1 - \rho^2) } = \frac{1}{3} \]

\[ \uparrow \]

utilizing \( AB^* + A^*B = 2 \text{Re} \left[ AB^* \right] \) and \( AB^* - A^*B = 2 \text{Im} \left[ AB^* \right] \), we can assume that \( A = \rho r(k)e^{j2\pi\epsilon} \) and \( B = r(k+N) \).
Derived of ML Estimator

\[ f(r(k), r(k + N)) = \exp \left( \frac{-|r(k)|^2 - 2 \rho \text{Re}[e^{j2\pi \varepsilon} r(k)r^*(k + N)] + |r(k + N)|^2}{(\sigma_s^2 + \sigma_n^2)(1 - \rho^2)} \right) \]

taking \( \langle 2 \rangle \) and \( \langle 3 \rangle \) into \( \langle 1 \rangle \) ==> 

\[
\sum_{k=\theta}^{\theta+L-1} \log \left( \frac{f(r(k), r(k + N))}{f(r(k))f(r(k + N))} \right)
\]

\[
= \sum_{k=\theta}^{\theta+L-1} \log \left( \frac{\pi^2 \left( \sigma_s^2 + \sigma_n^2 \right)^2}{\exp \left( -\left| r(k) \right|^2 - 2 \rho \text{Re}[e^{j2\pi \varepsilon} r(k)r^*(k + N)] + \left| r(k + N) \right|^2 \right)} \frac{1}{\left( \sigma_s^2 + \sigma_n^2 \right)(1 - \rho^2)} \right)
\]
Derived of ML Estimator

\[
\sum_{k=\theta}^{\theta+L-1} \log \left\{ \exp \left( \frac{\left( \left| r(k) \right|^2 + \left| r(k+N) \right|^2 \right) \left( 1 - \rho^2 \right) - \left| r(k) \right|^2 + 2 \rho \text{Re} \left[ e^{j2\pi \rho} r(k) r^*(k+N) \right] - \left| r(k+N) \right|^2}{\left( \sigma_s^2 + \sigma_n^2 \right) \left( 1 - \rho^2 \right)} \right) \left( 1 - \rho^2 \right) \right\} = C_1
\]

\[
= C_2 \sum_{k=\theta}^{\theta+L-1} \left\{ \text{Re} \left[ e^{j2\pi \rho} r(k) r^*(k+N) \right] - 0.5 \rho \left( \left| r(k) \right|^2 + \left| r(k+N) \right|^2 \right) \right\} + C_1
\]

\[
= C_1 + C_2 \left[ r(\theta) \cos(2\pi \varepsilon + \angle r(\theta)) - \rho \Phi(\phi) \right]
\]

where \( C_1 = \sum_{k=\theta}^{\theta+L-1} - \log \left( 1 - \rho^2 \right) \), and \( C_2 = \frac{2 \rho}{\left( \sigma_s^2 + \sigma_n^2 \right) \left( 1 - \rho^2 \right)} \)
Abstract

One of the widely used synchronization algorithms is the maximum likelihood (ML) estimation algorithm in last paper. However, this algorithm does not fully use the correlation of the cyclic prefix samples and the useful data samples in the observed samples, resulting in suboptimal performance. In this Letter we present the truly optimal ML estimator of the time and frequency offset for a general observation window size. The optimal ML estimator fully exploits the correlation residing in the cyclic prefix samples and the useful data samples.
Although this estimator was derived with the assumption that a receiver observes only $N_t+N$ samples, it requires the observation of $2N_t-1$ consecutive samples for *proper operation*, as can be seen from (6), (8) and (9).

\[ N_g \quad N \]

\[ N_t \quad \quad 2N+L=N_t+N \]
Optimal Maximum Likelihood Estimator

Fig. 1 Effect of delay on number of correlated samples in observation window
Fig. 2 Mean-square error of time offset estimate for optimal and suboptimal ML estimators for varying time offset ($N = 64$, $N_o = 16$)
Simulation

![Graph showing mean-square error of time offset estimate for optimal and suboptimal ML estimators for varying SNR (N = 64, N_g = 16)]

**Fig. 3** Mean-square error of time offset estimate for optimal and suboptimal ML estimators for varying SNR ($N = 64$, $N_g = 16$)