Code Tracking Loops
Introduction

- Code synchronization is generally carried out in two steps:
  - **Acquisition (coarse synchronization)**: To aligns the incoming signal and the local PN signal to within one chip or less.
  - **Tracking (fine synchronization)**: To bring the difference of the two phases to zero.

- In most situation, the PN acquisition is performed before (or at best concurrently with) the carrier recovery and tracking.

- Note that the PN tracking and carrier tracking are continuous operation after the PN phase has been acquired because (a) there is relative motion between the transmitter and the receiver, and (b) imperfect oscillators.
Introduction

- Code tracking is accomplished using **phase-locked techniques**, very similar to those used for generation of coherent carrier references.
- The principal difference between the phase-locked loops used for carrier tracking and code tracking is in the implementation of the phase discriminator.
- For carrier tracking, the discriminator is often as simple as a multiplier, whereas for modern code tracking loops, several multipliers and usually pairs of filters and envelope detector will be employed in the phase discriminator.
- A tracking circuit generally consists of a feedback loop that monitor the error and adjusts the desired signal in such a way that the error goes to zero.
Introduction

- The loop bandwidth will be selected to be a compromise between a wide bandwidth, which facilitates tracking the dynamics of transmission delay, and a narrow bandwidth, which minimizes the tracking jitter due to interference.

- A convenient measure of the tracking performance is the synchronization error variance (tracking jitter), which should be made small.

- There are two common methods used for tracking:
  - *Delay-Lock loop (DLL)* tracking method.
    - A tracking loop that makes use of two independent correlators.
  - *Tau-Dither Loop (TDL)* tracking method.
    - A tracking loop that time shares a single correlator.
Timing jitter

A result that is particularly important to the spread-spectrum system designer is the relationship between the tracking jitter and the received signal-to-noise ratio in the loop bandwidth.

The tracking jitter is defined as

\[ \delta \triangleq \frac{\hat{\tau} - \tau}{T_c} \]

The rms tracking jitter is

\[ \sigma_\delta^2 = \int_{-\infty}^{\infty} S_{n^*}(f) |H(j2\pi f)|^2 df \]

where \( S_{n^*}(f) \) is the power spectrum of the Gaussian noise process at the input to the loop filter, and \( H(s) \) is the closed-loop transfer function defined:

\[ H(s) \triangleq \frac{\hat{\tau}(s)}{\tau(s)} \]
It has been shown that the optimum tracking discriminator for an arbitrary wideband signal received with additive white Gaussian noise (AWGN) is a multiplier that forms the product of the received signal plus noise and the first derivative with respect to time of the receiver generated replica of the transmitted signal. This discriminator is optimum in that its output is a maximum likelihood estimate of the phase difference between the two wideband signals in an AWGN environment. This means that the output phase error estimate is the most probable phase error, given the available received information.
The optimum tracking discriminator for an arbitrary wideband signal received with AWGN:

\[ r(t) = s(t - T_d) + n(t) \]

Diagram:
- **Differentiator** \( \frac{d}{dt} \) with gain \(-\frac{1}{\alpha} \) s/V
- **Lowpass filter**
- **Delay error estimate** \( \alpha (T_d - \hat{T}_d) \)
The received signal \( r(t) = s(t - T_d) + n(t) \) is multiplied by a differentiated and delay receiver-generated replica of \( s(t) \).

The multiplier output contains a dc component related to the delay error \( \hat{T}_d - T_d \), where \( \hat{T}_d \) is the estimate of the transmission delay.

This dc component is extracted by the lowpass filter and used to correct the delay of the voltage controllable delay line.

This tracking loop configuration is not commonly used in modern spread-spectrum systems and the operation of the loop will be described for a single special case.
Optimum Tracking of Wideband Signals

\[
\begin{align*}
\left\{ \begin{array}{l}
\hat{T}_d > T_d \\
|\hat{T}_d - T_d| < T_c
\end{array} \right. 
\end{align*}
\]
Operation of the code tracking loop for a special case:

- The received signal is shown in figure (a).
- The tracking loop is to produce the signal \( c(t - \hat{T}_d) \), shown in figure (b), with \( \delta = \left( T_d - \hat{T}_d \right) / T_c \) as small as possible.
- The first derivative of figure (b) with respect to time is shown in figure (c). The derivative is a series of impulse functions.
- The tracking loop multiplier output is shown in figure (d).
- The dc component of the multiplier output is the time average of

\[
c(t - T_d) \frac{d}{dt} \left[ c(t - \hat{T}_d) \right]
\]

which is \((N + 1)/NT_c\).
When $\hat{T}_d < T_c$ and $|T_d - \hat{T}_d| < T_c$, all of the impulses at the multiplier output are negative and the dc component is $-(N + 1)/NT_c$.

Optimum delay discriminator output dc component for m-sequence baseband tracking loop:

$$E \left[ c(t - T_d) \frac{d}{dt} c(t - T_d) \right]$$

$$\delta = \frac{T_d - \hat{T}_d}{T_c}$$
Then the delay line input will be positive and that this will decrease \( \hat{T}_d \) as required to drive delay error to zero.

When \( \hat{T}_d < T_d \) and \( |T_d - \hat{T}_d| < T_c \), all of the impulses at the multiplier output are negative and the dc component is \(- (N + 1)/NT_c\).

When \( |T_d - \hat{T}_d| \geq T_c \), there is an equal number of positive and negative multiplier output impulses, and the dc level is zero.

Whenever \( |T_d - \hat{T}_d| < T_c \), a voltage exists which pushes the delay \( \hat{T}_d \) in the correct direction.
Coherent Delay-Lock Tracking Loop

\[
\cos(2\pi f_c t + \theta)
\]

\[
\cos(2\pi f_c t + \theta)
\]

From carrier recovery system. From receiver output.

Delay-lock tracking loop with coherent carrier demodulation.
Coherent Delay-Lock Tracking Loop

- We assume that tracking is initiated after the acquisition circuit has brought the phase difference to within $\pm \Delta T_c$.
- We further assume without loss of generality that the signal $s(t) = \sqrt{2Pc(t)b(t)} \cos(2\pi f_c t + \theta)$ at the input of the receiver, i.e., the phase of $c(t)$ is zero.
- The local PN generator generates $c(t+\tau)$, where $|\tau|<\Delta T_c$. In addition, it also produces advanced and delayed version of PN signal: $c(t+\tau+\tau_d)$ and $c(t+\tau-\tau_d)$ for a fixed $\tau_d$.
- Then the advanced and delayed (early and late) PN signals are mixed (multiplied) with the incoming signal and the carrier $\cos(2\pi f_c t + \theta)$ from the carrier recovery circuit.
Coherent Delay-Lock Tracking Loop

The difference of two despread signals is an error signal which is feedback to adjust the phase of the PN generator.

The outputs from the mixers are

\[ \nu_1(t) = \sqrt{2P} c(t) c(t + \tau + \tau_d) b(t) \cos^2(2\pi f_c t + \theta) \]

\[ = \sqrt{\frac{P}{2}} c(t) c(t + \tau + \tau_d) b(t) [1 + \cos(4\pi f_c t + 2\theta)] \]

\[ \nu_2(t) = \sqrt{2P} c(t) c(t + \tau - \tau_d) b(t) \cos^2(2\pi f_c t + \theta) \]

\[ = \sqrt{\frac{P}{2}} c(t) c(t + \tau - \tau_d) b(t) [1 + \cos(4\pi f_c t + 2\theta)] \]

The lowpass filters have bandwidth wide enough to pass \( b(t) \), but narrow enough that it average (lowpass filters) the \( c(t)c(t+ \tau + \tau_d) \) component.
Therefore, after lowpass filtering and mixing with \( \hat{b}(t) \), the signals are

\[
u_1(t) \approx \sqrt{\frac{P}{2}} \frac{b(t)\hat{b}(t)}{NT_c} \int_{t-NT_c}^{t} c(t')c(t'+\tau+\tau_d)dt' = \sqrt{\frac{P}{2}} \hat{b}(t)b(t)R_c(\tau+\tau_d)
\]

\[
u_2(t) \approx \sqrt{\frac{P}{2}} \frac{b(t)\hat{b}(t)}{NT_c} \int_{t-NT_c}^{t} c(t')c(t'+\tau-\tau_d)dt' = \sqrt{\frac{P}{2}} \hat{b}(t)b(t)R_c(\tau-\tau_d)
\]

where \( R_c(\tau) \) is the normalized autocorrelation function of the PN signal, which has a period of \( NT_c \):

\[
R_c(\delta) \approx \begin{cases} 1 - \frac{|\delta|}{T_c}, & \text{if } |\delta-iNT_c| < T_c \text{ for some integer } i \\ 0, & \text{otherwise} \end{cases}
\]
Assume that $\hat{b}(t) = b(t)$, so that $b(t)b(t) = 1$. The difference between $u_1(t)$ and $u_2(t)$ is

$$d(\tau) \equiv z(t)|_{\text{noise}=0} = u_2(t) - u_1(t) = \sqrt{\frac{P}{2}} [R_c(\tau - \tau_d) - R_c(\tau + \tau_d)]$$

where $\hat{b}(t)$ is an estimate of $b(t)$.

Estimation of $b(t)$ has some occasional errors, so the loop filter, which is a lowpass filter with a small bandwidth, averages out the product $\hat{b}(t)b(t)$.

Since the probability of error in a properly working receiver is small, the estimate $\hat{b}(t)$ equals $b(t)$ most of the time and the average of $b(t)b(t)$ is nearly 1.

The function $d(\tau)$ is called the *delay discriminator characteristic*, and it is also a periodic function with period $NT_c$. 
Coherent Delay-Lock Tracking Loop

Delay Discriminator Characteristic with $\tau_d / T_c = 0.25$.

![Delay Discriminator Characteristic (Tau_d / T_c = 0.25)](image_url)

- On Time
- Late
- Early
- S-Curve=Late-Early
Coherent Delay-Lock Tracking Loop

Delay Discriminator Characteristic with $\tau_d/T_c = 0.5$.

![Diagram of Delay Discriminator Characteristic with $\tau_d/T_c = 0.5$](image.png)

- On Time
- Late
- Early
- S-Curve=Late-Early

PN Code Period = 31
Coherent Delay-Lock Tracking Loop

Delay Discriminator Characteristic with $\tau_d / T_c = 0.75$. 

Delay Discriminator Characteristic (Tau_d / T_c = 0.75)

PN Code Period = 31

On Time
Late
-Early
S-Curve=Late-Early

Correlation

[ Tc ]
Coherent Delay-Lock Tracking Loop

- Delay Discriminator Characteristic with $\tau_d / T_c = 1.00$.

![Diagram showing delay discriminator characteristic with $\tau_d / T_c = 1.00$. The graph includes correlation values for 'On Time', 'Late', 'Early', and S-Curve=Late-Early with PN code period set at 31.]
Coherent Delay-Lock Tracking Loop

- Delay Discriminator Characteristic with $\tau_d / T_c = 0.25$.

![Graph showing the delay discriminator characteristic with PN code period of 4095. The graph includes red, blue, and green markers representing on time, late, and early, respectively, with an S-curve line connecting late and early markers.](image-url)
Coherent Delay-Lock Tracking Loop

Delay Discriminator Characteristic with $\tau_d / T_c = 0.5$.

Delay Discriminator Characteristic $(\tau_d / T_c = 0.50)$

- On Time
- Late
- Early
- S-Curve=Late-Early

PN Code Period = 4095
Coherent Delay-Lock Tracking Loop

Delays Discriminator Characteristic with $\tau_d / T_c = 0.75$.

Delay Discriminator Characteristic ($\tau_d / T_c = 0.75$)

PN Code Period = 4095

Correlation

On Time

Late

Early

S-Curve=Late-Early
Coherent Delay-Lock Tracking Loop

Delay Discriminator Characteristic with $\frac{\tau_d}{T_c} = 1.00$.

![Diagram showing the delay discriminator characteristic with PN code period 4095.](image-url)
For the purpose of discussion, suppose that \( y(t) = z(t) \).

The signal \( y(t) \) controls the voltage-controlled clock (VCC).

If \( y(t) = 0 \), the VCC and PN generator need no adjustment.

When \( y(t) \neq 0 \), the phase \( \tau \) of the local PN generator is adjusted appropriately (by delaying or advancing the phase, depending on whether \( y(t) \) is positive or negative).

The phase \( \tau \) of the PN generator must be increased when \( y(t) > 0 \) and decreased when \( y(t) < 0 \).
Noncoherent Delay-Lock Tracking Loop

Delay-lock tracking loop with noncoherent demodulation.
There are two differences between coherent and noncoherent DLL:

- A square-law envelop detector is used to eliminate the carrier.
- According to the square-law detector, we don’t need \( \hat{b}(t) \).

Since the signal \( s(t) \) and the noise \( n(t) \) is the same as above, the signal at the input of the upper bandpass filter is

\[
w_1(t) = \sqrt{2Pc(t)c(t + \tau + \tau_d)}b(t)\cos(2\pi f_c t + \theta) + \text{noise}
\]

Let the bandpass filters have a passband centered at \( \pm f_c \) and a bandwidth of 2B Hz and unity gain.

The bandwidth is wide enough to pass \( b(t) \), but narrow enough that the PN signal gets averaged out.
Noncoherent Delay-Lock Tracking Loop

With these, we have

\[ \nu_1(t) = \sqrt{2PR_c(\tau + \tau_d)}b(t)\cos(2\pi f_c t + \theta) + \text{noise} \]

where the noise is a bandpass noise.

The signal \( \nu_1(t) \) is passed through the square-law envelop detector, it is squared and lowpass filtered:

\[ u_1(t) = PR_c^2(\tau + \tau_d) + \text{noise} \]

where the noise is now a lowpass noise. Similarly,

\[ u_2(t) = PR_c^2(\tau - \tau_d) + \text{noise} \]

Therefore, the input to the loop filter is

\[ z(t) = u_2(t) - u_1(t) = P\left[R_c^2(\tau - \tau_d) - R_c^2(\tau + \tau_d)\right] + \text{noise} \]
The delay discriminator characteristic is the signal component of \( z(t) \): 

\[
d(\tau) = z(t) \big|_{noise=0} = P \left[ R_c^2(\tau - \tau_d) - R_c^2(\tau + \tau_d) \right]
\]

with

\[
d(\tau) = \begin{cases} 
0 & \text{for } -NT_c/2 < \tau \leq -T_c - \tau_d \\
- P \left( 1 + \frac{\tau + \tau_d}{T_c} \right)^2 , & \text{for } -T_c - \tau_d < \tau \leq -T_c + \tau_d \\
- P \left( \frac{4\tau_d}{T_c} \left( 1 + \frac{\tau}{T_c} \right) \right) , & \text{for } -T_c + \tau_d < \tau \leq -\tau_d \\
- P \left( \frac{4\tau_d}{T_c} \left( 1 - \frac{\tau_d}{T_c} \right) \right) , & \text{for } -\tau_d < \tau \leq \tau_d \\
- P \left( \frac{4\tau_d}{T_c} \left( 1 - \frac{\tau}{T_c} \right) \right) , & \text{for } \tau_d < \tau \leq T_c - \tau_d \\
- P \left( \frac{4\tau_d}{T_c} \left( 1 - \frac{\tau - \tau_d}{T_c} \right) \right) , & \text{for } T_c - \tau_d < \tau \leq T_c + \tau_d \\
- P \left( 1 - \frac{\tau - \tau_d}{T_c} \right) , & \text{for } T_c + \tau_d < \tau < NT_c/2 \\
\end{cases}
\]
Delay Discriminator Characteristic with $\tau_d / T_c = 0.25$.

Delay Discriminator Characteristic (Tau$_d$ / T$_c$ = 0.25)

PN Code Period = 4095

On Time
Late
-Early
S-Curve=Late-Early
Delay Discriminator Characteristic with $\tau_d / T_c = 0.45$. 

Noncoherent Delay-Lock Tracking Loop

Delay Discriminator Characteristic (Tau_d / T_c = 0.45)
Delay Discriminator Characteristic with $\frac{\tau_d}{T_c} = 0.75$.
Delay Discriminator Characteristic with $\frac{\tau_d}{T_c} = 1.00$. 

Noncoherent Delay-Lock Tracking Loop
One problem with the delay lock loop is that the two branches in this loop must have matched characteristics (gain balance). Otherwise, the loop will be locked to a non-zero point, say $\tau_1$.

One method to solve this question is by using the tau-dither tracking loop.

Other methods may be found, for example, in:

Tau-Dither Tracking Loop

- Tau-Dither tracking loop (or *time-shared early-late tracking loop*) uses only correlator branch.
The tau-dither loop eliminates the problem of matching the characteristics of the two correlator branches. However, the signal power in the tau-dither loop is 3 dB smaller than that in the delay-lock loop and the tracking jitter is larger in the tau-dither loop.

The dithering is controlled by two pulse signals $q_1(t)$ and $q_2(t)$, which alternately let the advanced and delayed versions of the PN signal through at a rate of $F_D=1/T_D$, where $T_D \gg T_c$.

The effect of correlation with early and late versions of the PN signal is achieved by dithering back and forth between these early and late signals.

The square wave $q(t)$ also controls the signal into the loop filter, in synchronism with the phase dithering.
Tau-Dither Tracking Loop

Dithering Loop Switching Functions

$q(t)$

$q_1(t)$

$q_2(t)$
The signal \( w(t) \) is, alternately, equal to \( w_1(t) \) and \( w_2(t) \), where
\[
\begin{align*}
  w_1(t) &= s(t)c(t + \tau + \tau_d) + \text{noise} \\
  w_2(t) &= s(t)c(t + \tau - \tau_d) + \text{noise}
\end{align*}
\]

Using \( q_1(t) \) and \( q_2(t) \), we can write
\[
w(t) = q_1(t)w_1(t) + q_2(t)w_2(t) + \text{noise}
\]

And the signal \( u(t) \) alternates between \( u_1(t) \) and \( u_2(t) \), where
\[
\begin{align*}
  u_1(t) &= PR_c^2(\tau + \tau_d) + \text{noise} \\
  u_2(t) &= PR_c^2(\tau - \tau_d) + \text{noise}
\end{align*}
\]

Then, we have
\[
\begin{align*}
  z(t) &= q(t)u(t) = q(t)[q_1(t)u_1(t) + q_2(t)u_2(t)] \\
        &= q_1(t)u_1(t) - q_2(t)u_2(t) \\
        &= P\left[q_1(t)R_c^2(\tau + \tau_d) - q_2(t)R_c^2(\tau - \tau_d)\right] + \text{noise}
\end{align*}
\]
If the loop filter has a narrow bandwidth, compared to $f_D$, the signal $q_1(t)$ and $q_2(t)$ are effectively averaged, yielding the delay discriminator characteristic of

$$d(\tau) = P\left\{ \text{Ave.}[q_1(t)]R_c^2(\tau + \tau_d) - \text{Ave.}[q_2(t)]R_c^2(\tau - \tau_d) \right\}$$

$$= \frac{P}{2} \left[ R_c^2(\tau + \tau_d) - R_c^2(\tau - \tau_d) \right]$$

which is half of that for the delay-lock loop.

Therefore, the signal power of the tau-dither loop is 3 dB smaller than that of the delay-lock loop.
References