Spread Spectrum Signal for Digital Communications
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- Multiple Access Schemes
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- Generation of Pseudo-Noise (PN) Sequences
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Multiple Access Schemes
Multiple Access Schemes

- Time Division Multiple Access (TDMA)
- Frequency Division Multiple Access (FDMA)
- Space Division Multiple Access (SDMA)
- Code Division Multiple Access (CDMA)
Multiple Access -- TDMA

- Partition the time axis into frame of $n$ slots and assign slots in some fashion
  - require synchronization between users.
  - allow variable rate sources (e.g. assign multiple slots per frame to a user).

![Diagram of time slot allocation](Diagram.png)
Multiple Access -- FDMA

Partition the spectrum into a set of bands and assign a band to each user

- no-need for synchronization in time between users
- different RF carrier frequencies
- variable peak power in the total signal
- inflexible to variable data rate per terminal
- the idle channel cannot be used by other users to increase or share capacity
- low complexity to implement

Frequency Orthogonality!!
FDMA Channels
TDMA Channels on Multiple Carrier Frequencies
TDMA with Use of Frequency Hopping Technique

Add the frequency diversity by frequency hopping to reduce the frequency-selective interference.
Multiple Access -- SDMA

- Space Division Multiple Access (SDMA) serves different users by using spot beam antennas.
- These different areas covered by the antenna beam may be served by the same frequency (in a TDMA or CDMA system) or different frequencies (in an FDMA system).
- Use array antenna to separate the simultaneously received signals of spatially separated subscribers by exploiting the directional selectivity of the mobile radio channel.
- The SDMA technique can be combined with each of the other multiple access techniques (FDMA, TDMA, CDMA) to increase the network capacity.
Multiple Access -- SDMA

- $d_1(t)$ enters the modulator to modulate signal $d_1(t)$.
- $d_2(t)$ enters the modulator to modulate signal $d_2(t)$.
- Signals from MS 1 and MS 2 are transmitted to BTS.
- BTS demodulates and separates the signals.
- Outputs: $d_1(t)$ and $d_2(t)$.
Spread Spectrum Communications
Spread Spectrum Communications

Definition: The transmitted signal must occupy a bandwidth which is large than the information bit rate and which is independent of the information bit rate.

Demodulation must be accomplished, in part, by correlation of the received signal with a replica of the signal used in the transmitter to spread the information signal.

Possess pseudo-randomness, which makes the signals appear similar to random noise & difficult to demodulate by receivers other than the intended ones.
Spread Spectrum Communications

**Advantages**

- Jam resistance
- Low probability of intercept
- Resistance to multi-path fading
- Frequency sharing
- Channel sharing, soft capacity, soft blocking
- Soft handoff

**Disadvantages**

- Self-jamming
- Near-far problem
- Implementation is more complex
**Techniques for Spread Spectrum - 1**

- **Direct Sequence Spread Spectrum (DSSS)**
  - A carrier is modulated by a digital code in which the code bit rate is much larger than the information signal bit rate. These systems are also called *pseudo-noise systems*.
Time-hopped Spread Spectrum (THSS)

- The transmission time is divided into intervals called frames. Each frame is divided into time slots. During each frame, one and only one time slot is modulated with a message.
Techniques for Spread Spectrum - 3

**Frequency Hopping Spread Spectrum (FHSS)**

- The carrier frequency is shifted in discrete increments in a pattern generated by a code sequence.
- *Fast-hop*: frequency hopping occurs at a rate that is greater than the message bit rate.
- *Slow-hop*: the hop rate is less than the message bit rate.
Idealized Model of Baseband Spread-Spectrum System (DSSS System)

(a) Transmitter

(b) Channel

(c) Receiver
Waveforms in the Transmitter of DSSS

data signal

spreading code

baseband spread spectrum signal
DSSS Technique in the Passband
- Coherent Binary Phase-Shift Keying

Transmitter
DSSS Technique in the Passband
- Coherent Binary Phase-Shift Keying

Receiver
Power Spectral Density

Processing Gain: \( N = \frac{T}{T_c} \)
Power Spectral Density Relative to Narrow Band Interference (NBI)

Power spectral densities in the channel

- Narrow Band Interference
- Bandpass SS Signal

\[ \frac{2\pi}{\omega_c} \]

Frequency
Power Spectral Density After Despreading

Despreading

Input to the integrator (or low pass filter)

- Rejected interference
- Detector noise
- Spread interference
- Low pass filter (Integrator)
- Collapsed data signal

Frequency: $\frac{1}{T}$ to $\frac{1}{T_c}$
Synchronization

- For proper operation, a spread-spectrum communication system requires that the locally generated PN sequence used in the receiver to despread the received signal be synchronized to the PN sequence used to spread the transmitted signal in the transmitter.

- A solution to the synchronization problem consists of two parts: acquisition and tracking.

  - In acquisition, or coarse synchronization, the two PN codes are aligned to within a fraction of the chip in as short a time as possible.
  
  - Once the incoming PN code has been acquired, tracking, or fine synchronization, takes place.
Typically, PN acquisition proceeds in two steps:

- The received signal is multiplied by a locally generated PN code to produce a measure of correlation between it and the PH code used in the transmitter.
- An appropriate decision-rule and search strategy is used to process the measure of correlation so obtained to determine whether the two codes are in synchronism and what to do if they are not.

For tracking, it is accomplished using phase-lock techniques, similar to those used for the local generation of coherent carrier references.
Generation of Pseudo-Noise (PN) Sequences
Contents

- Hadamard Codes
- Systematic Linear Binary Block Codes
- Cyclic Codes
- Maximum-Length Shift-Register Codes (*m*-sequence)
- Preferred Sequences
- Gold Sequences
Hadamard Codes

- Hadamard code is obtained by selecting the rows of a Hadamard matrix.
- A Hadamard matrix $M_n$ is an $n \times n$ matrix that any row differs from any other row in exactly $n/2$ positions.

$$M_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M_{2n} = \begin{bmatrix} M_n & M_n \\ M_n & \bar{M}_n \end{bmatrix}$$

$M_n$ and $\bar{M}_n$ form a linear binary code of block length $n$.

$$M_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

We can generate Hadamard codes with block length $n = 2^m$, and $d_{\text{min}} = \frac{1}{2} n = 2^{m-1}$, where $m$ is a positive integer.
Example of Hadamard Codes

**Hadamard Code of Length 8**

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
\end{bmatrix}
\]
Correlation of Orthogonal Codes

- Correlation properties of orthogonal codes are very sensitive to synchronization.
- Orthogonality of OVSF codes (or Hadamard codes) is achieved when codes are synchronized.
- Orthogonality of OVSF codes (or Hadamard codes) may not be maintained when codes are not synchronized.
Orthogonality is achieved when synchronization is maintained.
Orthogonality is maintained when codes are not synchronized
Orthogonality isn’t maintained when codes are not synchronized.
Pseudo-Noise Sequences

From Wikipedia, the free encyclopedia

In cryptography, **pseudorandom noise** (PRN) is a signal similar to noise which satisfies one or more of the standard tests for statistical randomness.

Although it seems to lack any definite pattern, pseudorandom noise consists of a deterministic sequence of pulses that will repeat itself after its period.

In cryptographic devices, the pseudorandom noise pattern is determined by a key and the repetition period can be very long, even millions of years.
Pseudo-Noise Sequences

In spread-spectrum systems, the receiver correlates a locally generated signal with the received signal. Such spread-spectrum systems require a set of one or more "codes" or "sequences" such that

- Like random noise, the local sequence has a very low correlation with any other sequence in the set, or with the same sequence at a significantly different time offset, or with narrowband interference, or with thermal noise.
- Unlike random noise, it must be easy to generate exactly the same sequence at both the transmitter and the receiver, so the receiver's locally generated sequence has a very high correlation with the transmitted sequence.
Pseudo-Noise Sequences

- In a direct-sequence spread spectrum system, each bit in the pseudorandom binary sequence is known as a *chip* and the *inverse* of its period as *chip rate*. Compare bit rate and baud.

- In a frequency-hopping spread spectrum sequence, each value in the pseudorandom sequence is known as a *channel number* and the *inverse* of its period as the *hop rate*. FCC Part 15 mandates at least 50 different channels and at least a 2.5 Hz hop rate for narrowband frequency-hopping systems.

- A *pseudonoise code* (PN code) is one that has a spectrum similar to a random sequence of bits but is deterministically generated. The most commonly used sequences in direct-sequence spread spectrum systems are maximal length sequences, Gold codes, Kasami codes, and Barker codes.
A *pseudo-noise (PN) sequence* is a periodic binary sequence with a noiselike waveform that is usually generated by means of a feedback shift register.
A feedback shift register consists of an ordinary shift register made up of \( m \) flip-flop (two-state memory stages) and a logic circuit that are interconnected to form a feedback circuit.

With a total number of \( m \) flip-flops, the number of possible states of the shift register is at most \( 2^m \).

A feedback shift register is said to be linear when the feedback logic consists entirely of modulo-2 adders.

The all-zero state is not permitted. As a result, the period of a PN sequence produced by a linear feedback shift register with \( m \) flip-flops can’t exceed \( 2^m - 1 \).

When the period is exactly \( 2^m - 1 \), the PN sequence is called a maximal-length-sequence or simply \( m \)-sequence.
Properties of Maximal-Length Sequences

- **Balance property:** In each period of a maximal-length sequence, the number of 1s is always one more than the number of 0s.

- **Shift-and Add Property:** The modulo-2 sum of an $m$-sequence and any phase shift of the same sequence is another phase of the same $m$-sequence.

- If a window of width $m$ is slid along the sequence for $N$ shifts, each $m$-tuple except the all zero $m$-tuple will appear exactly once.
Properties of Maximal-Length Sequences

- **Run Property:**
  - By a “run”, we mean a subsequence of identical symbols (1s or 0s) within one period of the sequence.
  - Among the runs of 1s and of 0s in each period of a maximal-length sequence, one-half the runs of each kind are of length one, one-fourth are of length two, one-eighth are of length three, and so on as long as these fractions represent meaningful numbers of runs.

- **Autocorrelation property:** the autocorrelation function of a maximum-length sequence is periodic and binary-valued.
Properties of Maximal-Length Sequences

- Convert 0 $\rightarrow$ 1 and 1 $\rightarrow$ -1. Let $c(t)$ denote the resulting waveform of the maximal-length sequence. The period of the waveform $c(t)$ is: $T_b = NT_c$, where $T_c$ is the duration assigned to symbol 1 or 0 and $N = 2^m - 1$.

The autocorrelation function of a periodic signal $c(t)$ of period $T_b$ is:

$$R_c(\tau) = \frac{1}{T_b} \int_{-T_b/2}^{T_b/2} c(t) c(t - \tau) \, dt$$
Properties of Maximal-Length Sequences

The autocorrelation function of the maximal-length sequence is:

\[ R_c(\tau) = \begin{cases} 
1 - \frac{N + 1}{NT_c} |\tau| & |\tau| \leq T_c \\
-\frac{1}{N} & \text{otherwise}
\end{cases} \]
Properties of Maximal-Length Sequences

Periodicity in the time domain is transformed into uniform sampling in the frequency domain.

\[
S_c(f) = \frac{1}{N^2} \delta(f) + \frac{1 + N}{N^2} \sum_{n=-\infty}^{\infty} \text{sinc}^2 \left( \frac{n}{N} \right) \delta \left( f - \frac{n}{NT_c} \right)
\]
Example of Maximum-Length Shift-Register Codes

Three-stage \((m = 3)\) shift register with feedback.

<table>
<thead>
<tr>
<th>Information bits</th>
<th>Code words</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>0 0 1 1</td>
</tr>
<tr>
<td>0 1 0</td>
<td>0 1 0 0</td>
</tr>
<tr>
<td>0 1 1</td>
<td>0 1 1 0</td>
</tr>
<tr>
<td>1 0 0</td>
<td>1 0 0 1</td>
</tr>
<tr>
<td>1 0 1</td>
<td>1 0 1 0</td>
</tr>
<tr>
<td>1 1 0</td>
<td>1 1 0 0</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1 1 0 0</td>
</tr>
</tbody>
</table>

Maximum-length shift-register code for \(m = 3\)

Systematic Code
Maximum-Length Shift Register
Maximal Length Shift Register \((m=5)\)
Maximal Length Shift Register ($m=5$)
## Maximal-Length Sequences of Shift-Register Lengths (2-8)

<table>
<thead>
<tr>
<th>Shift-Register Length, m</th>
<th>Feedback Taps</th>
</tr>
</thead>
<tbody>
<tr>
<td>2*</td>
<td>[2, 1]</td>
</tr>
<tr>
<td>3*</td>
<td>[3, 1]</td>
</tr>
<tr>
<td>4</td>
<td>[4, 1]</td>
</tr>
<tr>
<td>5*</td>
<td>[5, 2], [5, 4, 3, 2], [5, 4, 2, 1]</td>
</tr>
<tr>
<td>6</td>
<td>[6, 1], [6, 5, 2, 1], [6, 5, 3, 2]</td>
</tr>
<tr>
<td>7*</td>
<td>[7, 1], [7, 3], [7, 3, 2, 1], [7, 4, 3, 2], [7, 6, 4, 2], [7, 6, 3, 1], [7, 6, 5, 2], [7, 6, 5, 4, 2, 1], [7, 5, 4, 3, 2, 1]</td>
</tr>
<tr>
<td>8</td>
<td>[8, 4, 3, 2], [8, 6, 5, 3], [8, 6, 5, 2], [8, 5, 3, 1], [8, 6, 5, 1], [8, 7, 6, 1], [8, 7, 6, 5, 2, 1], [8, 6, 4, 3, 2, 1]</td>
</tr>
</tbody>
</table>
Maximal-Length Sequences of Shift-Register Lengths (2-34)

<table>
<thead>
<tr>
<th>m</th>
<th>Stages connected to modulo-2 adder</th>
<th>m</th>
<th>Stages connected to modulo-2 adder</th>
<th>m</th>
<th>Stages connected to modulo-2 adder</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1, 2</td>
<td>13</td>
<td>1, 10, 11, 13</td>
<td>24</td>
<td>1, 18, 23, 24</td>
</tr>
<tr>
<td>3</td>
<td>1, 3</td>
<td>14</td>
<td>1, 5, 9, 14</td>
<td>25</td>
<td>1, 23</td>
</tr>
<tr>
<td>4</td>
<td>1, 4</td>
<td>15</td>
<td>1, 15</td>
<td>26</td>
<td>1, 21, 25, 26</td>
</tr>
<tr>
<td>5</td>
<td>1, 4</td>
<td>16</td>
<td>1, 5, 14, 16</td>
<td>27</td>
<td>1, 23, 26, 27</td>
</tr>
<tr>
<td>6</td>
<td>1, 6</td>
<td>17</td>
<td>1, 15</td>
<td>28</td>
<td>1, 26</td>
</tr>
<tr>
<td>7</td>
<td>1, 7</td>
<td>18</td>
<td>1, 12</td>
<td>29</td>
<td>1, 28</td>
</tr>
<tr>
<td>8</td>
<td>1, 5, 6, 7</td>
<td>19</td>
<td>1, 15, 18, 19</td>
<td>30</td>
<td>1, 8, 29, 30</td>
</tr>
<tr>
<td>9</td>
<td>1, 6</td>
<td>20</td>
<td>1, 18</td>
<td>31</td>
<td>1, 29</td>
</tr>
<tr>
<td>10</td>
<td>1, 8</td>
<td>21</td>
<td>1, 20</td>
<td>32</td>
<td>1, 11, 31, 32</td>
</tr>
<tr>
<td>11</td>
<td>1, 10</td>
<td>22</td>
<td>1, 22</td>
<td>33</td>
<td>1, 21</td>
</tr>
<tr>
<td>12</td>
<td>1, 7, 9, 12</td>
<td>23</td>
<td>1, 19</td>
<td>34</td>
<td>1, 8, 33, 34</td>
</tr>
</tbody>
</table>


Maximum-length shift-register codes exist for any positive value of $m$. 
Problems with $m$-sequence

- Problems 1: jammer can determine the feedback connections by observing only $2m-1$ chips from the PN sequence.
- Solution 1: Output sequences from several stages of the shift register or the outputs from several distinct $m$-sequences are combined in a nonlinear sequence that is considerably more difficult for the jammer to learn.
- Solution 2: Frequently changing the feedback connections and/or the number of stages in the shift registers.
Problems with \textit{m-sequence}

- Problem 2: periodic cross correlation function between any pair of \textit{m}-sequences of the same period can have relatively large peaks.

- Although it is possible to select a small subset of \textit{m}-sequences that have relatively smaller cross correlation peak values, the number of sequences in the set is usually too small for CDMA applications.
Consider two PN sequences of period $2^7-1=127$, one feedback shift register has the feedback taps [7,1] and the other one has the feedback taps [7,6,5,4]. Both sequences have the same autocorrelation function.
## Peak Cross Correlation of $m$ Sequences and Gold Sequences

<table>
<thead>
<tr>
<th>$m$</th>
<th>$n = 2^m - 1$</th>
<th>Number of $m$ sequences</th>
<th>$\phi_{\text{max}}$</th>
<th>$\phi_{\text{max}}/\phi(0)$</th>
<th>$t(m)$</th>
<th>$t(m)/\phi(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>2</td>
<td>5</td>
<td>0.71</td>
<td>5</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>2</td>
<td>9</td>
<td>0.60</td>
<td>9</td>
<td>0.60</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>6</td>
<td>11</td>
<td>0.35</td>
<td>9</td>
<td>0.29</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>6</td>
<td>23</td>
<td>0.36</td>
<td>17</td>
<td>0.27</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>18</td>
<td>41</td>
<td>0.32</td>
<td>17</td>
<td>0.13</td>
</tr>
<tr>
<td>8</td>
<td>255</td>
<td>16</td>
<td>95</td>
<td>0.37</td>
<td>33</td>
<td>0.13</td>
</tr>
<tr>
<td>9</td>
<td>511</td>
<td>48</td>
<td>113</td>
<td>0.22</td>
<td>33</td>
<td>0.06</td>
</tr>
<tr>
<td>10</td>
<td>1023</td>
<td>60</td>
<td>383</td>
<td>0.37</td>
<td>65</td>
<td>0.06</td>
</tr>
<tr>
<td>11</td>
<td>2047</td>
<td>176</td>
<td>287</td>
<td>0.14</td>
<td>65</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>4095</td>
<td>144</td>
<td>1407</td>
<td>0.34</td>
<td>129</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Gold’s theorem:

Gold and Kasami proved that certain pairs of $m$ sequences of length $n$ (e.g. $g_1(X)$ and $g_2(X)$) exhibit a three-valued cross correlation function with values \{ -1, -t(m), t(m)-2 \} where:

$$t(m) = \begin{cases} 2^{(m+1)/2} + 1 & \text{odd } m \\ 2^{(m+2)/2} + 1 & \text{even } m \end{cases}$$

Two $m$ sequences of length $n$ with a periodic cross correlation function that takes on the possible values \{ -1, -t(m), t(m)-2 \} are called \textit{preferred sequences}.

The shift register corresponding to the product polynomial $g_1(X) \cdot g_2(X)$ will generate $2^{m+1}$ different sequences, with each sequence having a period of $2^m-1$. 
Golden Sequences

- From a pair of preferred sequences, say \( a = [a_1 \ a_2 \ a_3 \ \ldots \ a_n] \) and \( b = [b_1 \ b_2 \ b_3 \ \ldots \ b_n] \), we construct a set of sequences of length \( n \) by taking the modulo-2 sum of \( a \) with the \( n \) cyclicly shifted versions of \( b \) or vice versa.

- Thus, we obtain \( n \) new periodic sequences with period \( n = 2^m - 1 \). Together with the original sequences \( a \) and \( b \), we have a total of \( n+2 \) sequences, which are called \textit{Gold sequences}.

- With the exception of the sequences \( a \) and \( b \), the set of Gold sequences is not comprised of maximum-length shift-register sequences of length \( n \).

- The cross correlation function for any pair of sequences from the set of \( n+2 \) Gold sequences is three-valued with possible values \{ -1, -t(m), t(m)-2 \}.

- The off-peak autocorrelation function for a Gold sequence takes on values from the set \{ -1, -t(m), t(m)-2 \}. 

Generator for a Gold Sequence of Period 127
Cross-Correlation Function

Cross-correlation function of a pair of Gold sequences based on the two PN sequences [7,4] and [7,6,5,4].

![Graph showing cross-correlation function and delay τ]
RAKE Receiver
Architecture of RAKE Receiver

Despread RAKE Finger (1)

Despread RAKE Finger (2)

Despread RAKE Finger (L)

Delay

Channel Estimation

$G^*_i$

MRC

$\Sigma$
Combining Schemes
To utilize the advantages of diversity techniques, channel parameters are necessary to be estimated.
- Arrival time of each path, Amplitude, and Phase.

Maximal Ratio Combiner (MRC):
- The combiner that achieves the best performance is one in which each output is multiplied by the corresponding complex-valued (conjugate) channel gain.
- The effect of this multiplication is to compensate for the phase shift in the channel and to weight the signal by a factor that is proportional to the signal strength.
Maximum Ratio Combining (MRC)

MRC: $G_i = A_i e^{j\theta_i}$

- Coherent Combining
- Channel Estimation
- Best Performance
Maximum Ratio Combining (MRC)

Received Envelope: \( r_L = \sum_{l=1}^{L} G_l \cdot r_l \)

Total Noise Power: \( \sigma_n^2 = \sum_{l=1}^{L} |G_l|^2 \sigma_{n,l}^2 \)

\( SNR: \quad SNR_L = \frac{r_L^2}{2 \cdot \sigma_n^2} = \frac{\left| \sum_{l=1}^{L} G_l \cdot r_l \right|^2}{2 \cdot \sum_{l=1}^{L} |G_l|^2 \cdot \sigma_{n,l}^2} \)

Since \( \left| \sum_{l=1}^{L} G_l \cdot r_l \right|^2 = \left| \sum_{l=1}^{L} G_l \sigma_{n,l} \left( \frac{r_l}{\sigma_{n,l}} \right) \right|^2 \)
Maximum Ratio Combining (MRC)

Chebychev's Inequality:
\[
\left| \sum_{l=1}^{L} G_l \cdot r_l \right| ^2 \leq \left| \sum_{l=1}^{L} G_l \sigma_{n,l} \right| ^2 \cdot \left| \sum_{l=1}^{L} \frac{r_l}{\sigma_{n,l}} \right| ^2
\]

\[
SNR_L \leq \frac{1}{2} \frac{\sum_{l=1}^{L} |G_l \sigma_{n,l}|^2 \cdot \sum_{l=1}^{L} \left| \frac{r_l}{\sigma_{n,l}} \right|^2}{\sum_{l=1}^{L} |G_l|^2 \sigma_{n,l}^2} = \frac{1}{2} \sum_{l=1}^{L} \frac{|r_l|^2}{\sigma_{n,l}^2} = \sum_{l=1}^{L} SNR_l
\]

With equality hold: \( G_l \sigma_{n,l} = k \frac{r_l^*}{\sigma_{n,l}} \)

\( \Rightarrow \) Output SNR = Sum of SNRs from all branches @ \( G_l \propto r_l^* \)
Equal Gain Combining

- The equal gain combining only compensates the channel phase shift.

- The gain for the EGC is given by \( G_i = e^{-j\theta_i} \)

- Thus, EGC is simpler to implement than MRC. Moreover, no channel amplitude estimation is needed.
Orthogonality Restoring Combining (ORC)

- The orthogonality restoring combining compensates the channel phase shift and the channel amplitude fading.
- The gain for the ORC is given by
  \[ G_i = \frac{1}{A_i} e^{-j\theta_i} \]
- However, low level subcarriers tend to be multiplied by high gains, and the noise components are amplified at weaker subcarriers.
- The noise amplification effect degrades the BER performance.

\[ h_i = A_i e^{j\theta_i} \]

- \( h_i \) is the \( ith \) subchannel's channel response
- \( A_i \) is the magnitude of \( h_i \)
- \( \theta_i \) is the phase of \( h_i \)